Non-Profit Support in Education: Resource Allocation and Students' Lifetime Outcomes

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Problem Definition: One of the seventeen United Nations Sustainable Development Goals aims for inclusive and equitable quality education, with lifelong benefits, for all (United Nations 2023). Our work in this paper focuses on the operations of non-profit organizations (NPOs) that broaden access to high-quality education for underprivileged students. Specifically, we analyze the resource-allocation strategy of an NPO that adopts a two-stage structure in allocating resources to its beneficiaries; e.g., free pre-secondary education (first stage) for all underprivileged students in a target population, followed by sponsorships for post-secondary education (second stage) at leading institutions to those students who demonstrate commendable performance in the first stage. The lifetime outcomes of the beneficiaries depend on their own effort and on the quality of the resources that the NPO provides.

Methodology/Results: We adopt a principal-agent framework with moral hazard in the absence of monetary transfers. We establish the strategic role of an NPO's resource-allocation strategy on the effort beneficiaries invest and their lifetime outcomes – in particular, despite the supportive nature of the NPO's resources and despite possessing enough quantity of these resources to support all beneficiaries, we show why the NPO benefits from *deliberately throttling access* to the resources.

Managerial Implications: Our findings have important implications for the design of such support policies of NPOs. For a fixed endowment of resources, we demonstrate the effect of competition among the beneficiaries on their effort and lifetime outcomes. Likewise, for a fixed population of beneficiaries, we show the value of creating a *strategic scarcity* of resources for incentivizing beneficiaries to exert more effort. Finally, when faced with multiple beneficiary subgroups, we identify when the NPO benefits from *pooling* the beneficiary subgroups vs. earmarking dedicated resources for each subgroup.

Key words: Non-Profit Organizations, Social Sustainability, Resource Allocation, Moral Hazard.

"Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all."

— Goal #4 of the United Nations Sustainable Development Goals

https://sdgs.un.org/goals/goal4

1. Introduction

Nonprofit organizations (NPO's) worldwide have long played an important role in improving the lives of the underprivileged. Through a myriad of programs and initiatives, NPOs remain important institutions in our collective progress towards the United Nations' Sustainable Development Goals (SDGs; United Nations 2023). NPOs play an even greater role in developing countries, perhaps due to weaker state capacity (International Monetary Fund 2023). To mention just two domains in which NPOs have made commendable societal contributions towards achieving the UN's SDGs: (a) NPOs that operate in the delivery of healthcare, e.g., the Bill and Melinda Gates Foundation, offer a variety of programs that focus on improving health and learning outcomes among disadvantaged groups (https://www.gatesfoundation.org/our-work), (b) NPOs in the education sector, e.g., the Tata Trusts, focus on broadening access to high-quality education for underprivileged students (https://www.tatatrusts.org/our-work/education/broadening-access).

An important determinant of the effectiveness of an NPO is its resource-allocation strategy, i.e., how it assigns resources to its beneficiaries (de Véricourt and Lobo 2009, Feng and Shanthikumar 2016). This work is motivated by resource allocation among nonprofit initiatives in which the outcome to a beneficiary depends on their effort, such as in education. In particular, we focus on nonprofit initiatives – supported by an NPO – that adopt a *two-stage structure* in allocating resources to its beneficiaries: The first-stage support is standardized and is provided to all beneficiaries, while in the second stage, the quality of support provided to the beneficiaries is contingent on their first-stage performance. For instance, the first stage could be free K-12 education for all underprivileged students in a targeted population of beneficiaries and the second stage could be free post-secondary education – either at local universities with moderate admission criteria or at prestigious and highly competitive universities – contingent on the level of their first-stage performance. As we will see from the examples below, the two-stage structure is a reasonable abstraction of practice. However, in general, our analysis and insights remain qualitatively similar for a multi-stage structure as well.

We now discuss examples of well-known NPOs in the education sector that have adopted such a two-stage structure in their resource allocation.

Example 1: *Tata Trusts* is a prominent nonprofit organization supported by the Tata Group, a renowned business conglomerate with a presence in 80 countries. It also serves as an incubator for numerous nonprofit organizations, including the Karta Initiative, which operates in the education sector. This initiative strives to broaden access to high-quality education for underprivileged individuals from rural regions. To this end, the organization has established the Catalyst Scholarship Program (https://karta-initiative.org.in/index.php/why-karta/). The program's central purpose, as defined by the organization, is as follows:

> "We start working with young people when they turn 15, at the point when they are starting to think about their future. Our support helps them to navigate this period and make the most of post-school opportunities. But it doesn't stop there. We continue supporting exceptional students through to higher education at world-leading universities with our scholarship programme."

Here, note that the NPO (Tata Trusts) uses an exogenous process through which a target population of beneficiaries – namely, underprivileged students of age 15 from select rural regions – is identified. Typically, the size of this target population is chosen such that the NPO has an adequate amount of resources to support that size, if needed. The first stage of interaction between the NPO and the beneficiaries corresponds to the pre-secondary phase (i.e., K-12) during which the organization provides support to the chosen students. The NPO subsequently sponsors their post-secondary education, either at local institutions that are easy to get admitted to or at renowned and selective universities worldwide, depending on their performance in the pre-secondary phase. Thus, post-secondary education constitutes the second stage of interaction between the NPO and the students.

Example 2: The *D'Addario Foundation* in the United States is dedicated to providing support to underprivileged students pursuing education in music (https://foundation.daddario.com/why-support-us/). Through a range of programs, the foundation aims to empower these students and offer them opportunities for a brighter future in music. One notable initiative by the foundation is a college scholarship fund:

> "The nonprofit D'Addario Foundation established a college scholarship fund in 2018 to distribute merit and need-based scholarships to socio-economically disadvantaged youth. In order to receive a scholarship, a student must study music in a grantee organization for multiple years and show great promise but lack the resources to attend college or trade school."

Here, pre-college music education at grantee organizations supported by the foundation represents the first stage of interaction between the students and the foundation. Subsequently, to students who display exceptional ability in the first phase, the foundation grants scholarships for college education in the second stage of interaction.

Example 3: The *National Overseas Scholarship Scheme (NOSS)* of the Ministry of Social Justice and Empowerment, Government of India, aims to financially assist underprivileged students seeking higher education in top-ranked foreign institutes (https://nosmsje.gov.in/Default.aspx). The key objective of the initiative, as described by the ministry, is as follows:

> "The central sector scheme of National Overseas Scholarship is to facilitate low-income students belonging to the Scheduled Castes, Denotified Nomadic and Semi-Nomadic Tribes, Landless Agricultural Labourers, and Traditional Artisans category, to obtain higher education viz., Master degree or Ph.D courses, by studying abroad, thereby improving their economic and social status."

The government offers financial resources to candidates based on their academic performance and the ranking of the institution they gain admission to. The progress of the candidates is monitored through bi-annual reports for the continuation of the award in the future. In other words, the government employs an output-contingent allocation of resources to the candidates over multiple stages.

1.1. Research Questions and Main Results

To motivate our research questions, we use Tata Trusts' scholarship program as a running example in the discussion below. Consider an NPO that identifies a target population of beneficiaries through an exogenous process. For instance, for Tata Trusts, this target population consists of underprivileged 15-year-old students from select geographical areas. After the target population is identified, the NPO adopts a two-stage structure of evaluation (as described in the examples above) in its resource-allocation strategy. The quality of support provided to the beneficiaries during the first-stage of interaction between the NPO and the beneficiaries is standardized. In the context of the Tata Trusts, the quality of support that is provided to all students in the pre-secondary phase is identical. However, the second-stage support can vary in quality. For Tata Trusts, this is free college education provided either at an inexpensive, local university (a *base-quality* resource) or an expensive, globally recognized university (a *superior* resource). Figure 1 shows a broad schematic of the two-stage structure.

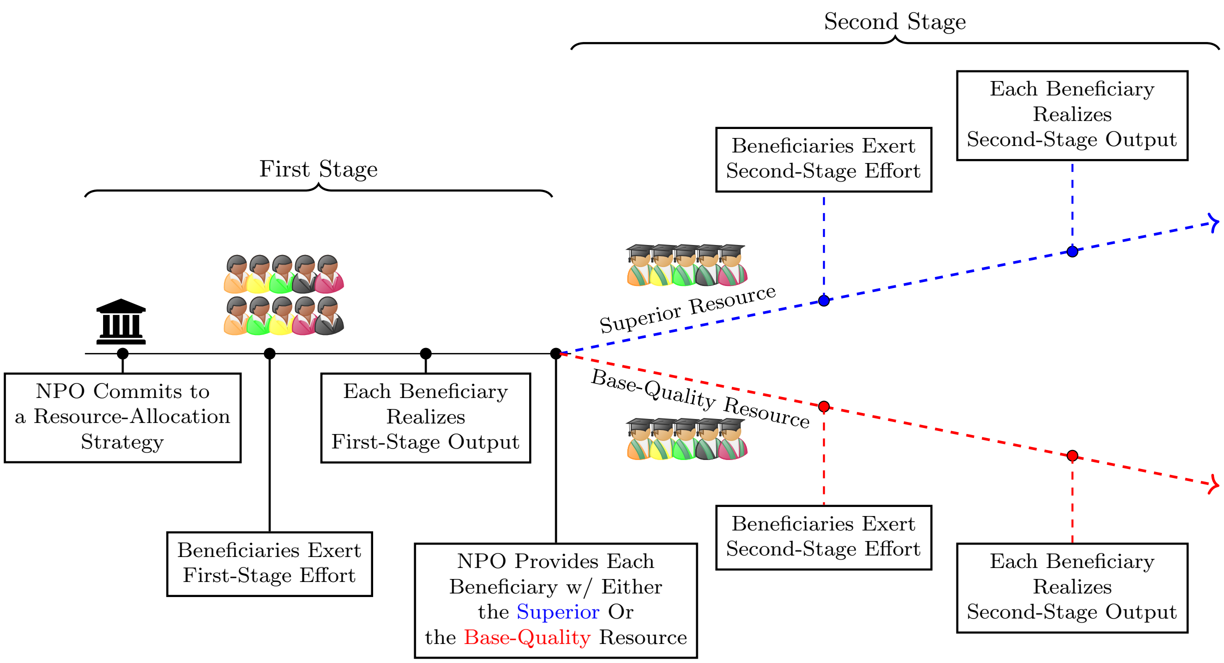

Figure 1 Schematic of the NPO's Two-Stage Structure for Allocating Resources to its Beneficiaries

The lifetime outcome of a beneficiary, a key component of the NPO's objective, is increasing in the beneficiary's effort in both stages. For example, Tata Trusts prefers that a student exert high effort both in the pre-secondary and post-secondary phases. Further, providing the beneficiary with the superior resource in the second-stage complements their effort and results in a better lifetime outcome as compared to providing the base-quality resource. For Tata Trusts, *ceteris paribus*, post-secondary education at a globally-recognized university results in a better lifetime outcome for a student as compared to that at a local, inexpensive university.

Analysis under Resource Adequacy: Summary of Results and Insights

Suppose that the NPO has an adequate quantity of the superior resource to support their entire target population of beneficiaries, if needed. We ask the following fundamental question:

(Q1) Despite possessing enough quantity of the superior resource for the entire target population of beneficiaries, should an NPO throttle access to the superior resource? In other words, should the NPO provide unrestricted access to the superior resource, or should the superior resource be provided contingent on a beneficiary's performance in the first stage?

The conventional wisdom to restricting access to the superior resource is its scarcity. Therefore, one might intuit that in the absence of scarcity of the superior resource, the NPO should provide them to all beneficiaries. Further, the NPO does not benefit from withholding the superior resource while the beneficiary strictly benefits from receiving the resource. Moreover, for any level of stage-2 effort, the beneficiary's output is higher with the superior resource, as compared to that with the base-quality resource. Despite these incentives, we show that the NPO benefits from deliberately restricting access to the superior resource in the absence of scarcity. By making the access of the superior resource contingent on the stage-1 output, the NPO induces a higher stage-1 output but a weaker stage-2 output (due to the possibility that the beneficiary might not always receive the superior resource), relative to always providing access to the superior resource. We show that this strategy leads to a superior outcome for both the NPO and the beneficiary, relative to that under unrestricted access. More generally, our work addresses the following question:

(Q2) What is the strategic role of an NPO's resource-allocation strategy on a beneficiary's effort and lifetime outcome?

From a theoretical standpoint, we adopt a principal-agent framework with moral hazard in the absence of monetary transfers. A resource-allocation strategy of an NPO (principal) is essentially defined by a menu of probability mass functions (with each such function also referred to as a *lottery*) over the resources, for every possible realization of the first-stage output. While the canonical principal-agent problem with moral hazard under a risk-neutral principal and a risk-neutral agent that allows for transfers is simple to solve (Bolton and Dewatripont 2004), ours is one that does not involve any monetary transfers. Further, unlike the canonical problem, the set of rewards (resources that the NPO possesses) is a discrete set. We show that among all possible resource-allocation strategies, a simple *threshold strategy* – where the NPO assigns the superior resource if and only if the beneficiary's first-stage output exceeds a threshold – is optimal. This understanding allows us to examine how a beneficiary's effort and lifetime outcome change with changes in the underlying environment.

Probing further, to understand the impact of the underlying environment (e.g., the beneficiary's cost of effort and the noise in the first stage outcome) on the NPO's optimal resource-allocation strategy, and thereby a beneficiary's lifetime outcome in equilibrium, we examine the following question:

(Q3) How does an increase in the beneficiary's cost of effort and the noise in the first-stage outcome affect the NPO's propensity to provide the beneficiary with the superior resource?

In the canonical principal-agent problem with moral hazard, an agent's effort cost or noise in the environment are considered to be frictions that make contracting less efficient. However in our setting, contrary to intuition, we show that the beneficiary is more likely to receive the superior resource if their cost of effort is higher or there is greater noise in the first-stage outcome. This is despite the fact that the beneficiary's effort is decreasing in the cost of effort and the noise in the environment. We discuss the underlying reasoning behind this property in Section 6.2.

In Section 3, we will define our two-stage model to address the above questions. Here, to emphasize that our model is firmly rooted in our context, namely education, we briefly highlight three aspects that the model captures and collectively distinguish it from other settings: (a) Students' early-stage academic efforts have a lifelong positive effect (Schweinhart 2003). (b) Students tend to overestimate the cost of their early-stage effort and fail to sufficiently recognize its lifetime impact (Bettinger and Slonim 2007, Kaur et al. 2010). The general phenomenon of overestimation of immediate costs is referred to as *hyperbolic discounting* in the Behavioral Economics literature (O'Donoghue and Rabin 1999, 2015). (c) Adequately valuing childhood happiness and reducing student stress are important factors that educational institutions should consider for achieving superior lifetime outcomes of their students. While providing access to education, nonprofits often fall short of adequately recognizing such factors, thereby under-emphasizing students' cost of effort (Clark et al. 2020, Seror 2022).

Analysis under Resource Scarcity: Summary of Results and Insights

Having analyzed the setting where the NPO has an adequate amount of superior resources to support its target population of beneficiaries, we turn to the case where an NPO may not have adequate resources, i.e., the case of *resource scarcity*. We ask the following question:

(Q4) How does scarcity of resources (equivalently, competition among the beneficiaries) affect beneficiaries' efforts and lifetime outcomes?

We find that the effort exerted by the beneficiaries is non-monotone in the extent of resource scarcity. Specifically, for low levels of scarcity, a beneficiary's effort is increasing in the extent of scarcity. However, for high levels of scarcity, agents tend to progressively "give up", i.e., an agent's effort is decreasing in the extent of scarcity. Interpreted differently, this observation highlights that, from the NPO's viewpoint, the optimal amount of scarcity is intermediate (neither too low nor too high). Our analysis helps us develop three valuable and actionable insights on how the NPO can benefit from engineering a *strategic scarcity* of resources:

- **Design of the Beneficiary-Population for a Fixed Endowment of Resources**: The role of scarcity and the manner in which competition moderates beneficiaries' effort has implications for how an NPO should choose its target population of beneficiaries. For a given endowment of resources, our results demonstrate that the NPO can induce superior outcomes by strategically choosing the size of the beneficiary-population to induce the highest effort from the beneficiaries.
- **Endowment of Resources and Strategic Scarcity**: For a given population of beneficiaries, we demonstrate the effect of the endowment of resources on the effort beneficiaries invest and their lifetime outcomes. The endowment of resources that induces the highest effort from the beneficiaries is neither too low nor too high; thus the NPO can benefit from allocating resources strategically in a manner so as to create an ideal level of scarcity of resources.
- **Pooled vs. Dedicated Resources for Multi-Beneficiary Pools**: NPOs often design initiatives for multiple pools of beneficiaries. For example, the Tata Trusts organization supports underprivileged students from multiple high schools in multiple states. A question that NPO managers, who design such initiatives, face is whether these beneficiary pools should be provided with dedicated resources (e.g., a dedicated number of resources per high school or state), or should all the beneficiaries be pooled together and provided with the pooled set of resources? Our results show that the choice depends on the extent of competition among the beneficiaries and resource scarcity. Under low levels of competition, the NPO is better off by managing the multiple beneficiary subgroups as one big pool. However, under high levels of competition, the NPO is better off earmarking dedicated resources for each beneficiary pool.

It will be clear from our ensuing analysis that the questions above are all part of the same chain, and understanding the resource-allocation is important to appreciate the strategic insights.

Extensions

We generalize our work in several directions to establish the robustness of the NPO's optimal resource-allocation strategy identified in our base model.

(a) Our base model analyzes the NPO's optimal resource-allocation strategy for the case of two (vertically differentiated) resources, viz., the superior resource and the base-quality resource, and the NPO has sufficient amount of resources. In Section 7, to generalize our results to the case of multiple (vertically differentiated) resources, we prove that the analysis under multiple (three or more) resources can be reduced to the case of two resources.

(b) Our base model assumes a population of homogenous beneficiaries. In Section 9, we establish the optimality of the threshold strategy for a population of heterogenous beneficiaries.

(c) Our base setting assumes an additive model for the beneficiary's output, where the output is the sum of the effort and a Gaussian noise term. In Appendix C, we generalize this model output to distributions that follow the Monotone Likelihood Ratio Property (MLRP).

2. Related Literature

There has been a growing interest within the OM community to address problems of social significance. In particular, there has been an emerging stream of work on understanding and addressing the challenges faced by NPOs. For comprehensive reviews of this stream, we refer the reader to Feng and Shanthikumar (2016) and Berenguer and Shen (2020). As discussed in both these papers, there are a myriad of challenges that NPOs face. Given our focus in this paper – resource allocation to beneficiaries in the context of education – we restrict attention to papers that are closer to our context.

First, we review the literature in non-profit OM that studies resource management and benefits allocation. de Véricourt and Lobo (2009) consider an NPO that also engages in for-profit activities to subsidize their mission activities, e.g., a hospital that operates a for-profit arm and a not-for-profit arm. They analyze how such an NPO should divide organizational resources to balance investment (revenue generated from for-profit activities) vs. consumption (social capital gained through their mission activities). Lien et al. (2014) consider an NPO that distributes a scarce resource, e.g., food, over time to meet their beneficiaries' needs, with an aim to balance equity and effectiveness of their service. Natarajan and Swaminathan (2014) and Natarajan and Swaminathan (2017) analyze the procurement of resources and their provision to beneficiaries when funding is limited, erratic, and unpredictable.

While the above papers take funding as a given, Devalkar et al. (2017) study a novel funding strategy for NPOs called "ex-post" funding, where, in addition to traditional (ex-ante) fundraising, an NPO also raises funds from donors who contribute based on the results (output) delivered by the NPO. In particular, they analyze the optimal mix of traditional and ex-post funds for a given project, and demonstrate its superiority over traditional fundraising. Sharma et al. (2021) analyze a closely related funding approach called payment for results, where social investors provide upfront funding to a capital constrained NPO. Based on the output generated, donors provide funding at the completion of the project and social investors are paid back.

In the context of healthcare products, Atasu et al. (2017) examine organizations that recover excess medical supplies and distribute these surpluses to underserved healthcare facilities in developing economies. Inspired by a recipient-driven model where recipients have access to full information on inventory and availability, they analyze the value of partial information disclosure and eliminating recipient competition. In the same realm, Zhang et al. (2020) adopt a mechanism design approach to select recipients for medical surplus based on their reported preferences. They show that withholding inventory information and eliciting preference rankings leads to greater value provision.

In contrast to the above papers that restrict attention to one project/initiative of an NPO, a separate stream of work has analyzed the collection (assortment) of services/initiatives that NPOs should provide. Zhang et al. (2022) study an innovative strategy where a NPO offers partially complete products or services to diverse beneficiaries with heterogenous needs. They show that this can emerge as a design strategy even in the absence of abundant resources. Arora et al. (2022) analyze an NPO's service portfolio and allocation of effort decisions under resource constraints, where potential beneficiaries do not have well-specified needs. While conventional wisdom suggests that a large portfolio of services helps meet the needs of all beneficiaries, they show that it is optimal for an NPO to offer fewer services and invest in advisory activities when there is greater heterogeneity in the needs of the beneficiaries.

Next, a large stream of work within the OM literature has analyzed how a firm (or service provider) should allot its scarce resources (limited inventory of goods and services) to a population of consumers. Given our focus, we review work that has analyzed resource allocation in the absence of monetary transfers. Swaminathan (2003) develops a decision support tool for distribution of scarce drugs free of charge across clinics and hospitals to balance efficiency, effectiveness and equity. Closer to our work, Gupta et al. (2023) analyze the role of non-monetary rewards in the presence of moral hazard to encourage an agent to induce effort over the long run. They demonstrate the performance of "limited-term" and "score-based" rewards in deterministic and stochastic environments. Several papers have adopted a mechanism design approach to ration scarce resources to recipients with heterogeneous valuations for the resources. Examples of this stream include Horner and Guo (2015), Balseiro et al. (2019), Gorokh et al. (2021), Gupta et al. (2024).

From a methodological standpoint, our work adopts a principal-agent framework in the presence of moral hazard. Contracting in the presence of moral hazard has been well studied in Economics and in OM. Due to the extensive nature of this stream, we avoid a formal review and direct the reader to a recent review paper by Georgiadis (2022).

We now proceed to define our base model, which studies the NPO's resource-allocation strategy for the case where the NPO has access to an adequate quantity of resources to support the entire population of beneficiaries, if needed. Later, in Section 8, we will discuss the case of resource scarcity, where the NPO has a limited quantity of resources.

3. Base Model: Allocation under Sufficient Amount of Resources

We consider the following two-period model under moral hazard, where a principal (NPO) incentivizes an agent (student) to exert costly effort, not via monetary transfer, but through the allocation of a superior resource. A principal has access to an abundance of two types of resources: a base-quality resource denoted by B, and a superior resource denoted by S. Let Φ denote the set of resource types; $\Phi = \{B, S\}$ (see Section 7 for an analysis with multiple resources, i.e., $|\Phi| \geq 2$). Let k_ϕ denotes the efficacy of the resource of type $\phi \in \Phi$. We assume that $k_S > k_B \geq 0$.

In period-1, the agent exerts effort, denoted by $e_1 \geq 0$, and produces a stochastic output y_1, where

$$y_1 = e_1 + \varepsilon_1, \tag{1}$$

where ε_1 is an idiosyncratic noise that is normally distributed with mean 0 and variance σ^2, i.e.,

$$\varepsilon_1 \sim \mathcal{N}\left(0, \sigma^2\right).$$

Let $\psi(\cdot|\mu)$ and $\Psi(\cdot|\mu)$ denote the p.d.f. and c.d.f. of the normal distribution with mean μ and variance σ^2. Further, let $\psi_{(-)}(\cdot|\mu)$ and $\psi_{(+)}(\cdot|\mu)$ denote the p.d.f. of the normal distribution with mean μ and standard deviation σ to the left and right of the mean, respectively; we note that these functions are invertible.

At the end of period-1, the principal provides a unit of a resource among the set Φ to the agent. Let $\phi \in \Phi$ denote the type of resource provided to the agent. In period-2, the agent exerts effort denoted by $e_2 \geq 0$ and produces a stochastic output y_2 as follows:

$$y_2 = k_\phi e_2 + \varepsilon_2, \tag{2}$$

where ε_2 is an independent zero-mean random variable, i.e., $\mathbb{E}[\varepsilon_2] = 0$. For any fixed e_2, observe that the superior resource leads to (stochastically) higher output than the base-quality resource.

Remark: Observe that in this model of the agent's period-1 output, $y_1|e_1 \sim \mathcal{N}(e_1, \sigma^2)$, the output can be negative. For settings that require a non-negative output, we can interpret y_t as a monotone transformation of the output, e.g., $y_t = \log(\text{Output}_t)$. Our analysis carries over in a straightforward manner to such settings. ■

Before proceeding further with the description of the model below, it is important to highlight the following three aspects we incorporate in terms of our application context, i.e., education:

(a) We model the agent's lifetime outcome as a function of the first- and second-stage efforts to capture the idea that a student's early-stage efforts have a life-long positive effect (Schweinhart 2003); see Section 3.5.

(b) We model the behavioral phenomenon that agents tend to exaggerate immediate costs relative to lifetime outcomes (referred to as *hyperbolic discounting* in the Behavioral Economics literature; see, e.g., O'Donoghue and Rabin 1999, 2015) by incorporating a cost-exaggeration factor in how they evaluate their payoff; see the factor β in (3) below. For instance, students often fail to sufficiently recognize the lifetime impact of their current academic efforts (Bettinger and Slonim 2007, Kaur et al. 2010).

(c) In education, a common theme of discussion in society is the necessity to value childhood happiness and reduce student stress (Clark et al. 2020, Seror 2022). There are instances of nonprofits under-appreciating the efforts of beneficiaries (Threlfall et al. 2013). We incorporate this theme by including a factor in the principal's utility function to measure the extent to which the principal under-emphasizes a student's cost of effort; see the factor γ in (4) below.

3.1. Payoffs

The agent's period-t payoff, $t \in \{1,2\}$, denoted by $\tilde{u}_t$, is as follows:

$$\tilde{u}_t(y_t, e_t) = y_t - \beta\left(\frac{c}{2}e_t^2\right), \tag{3}$$

where $\beta \geq 1$ captures the agent's exaggeration of his immediate costs. Alternatively, we can model the agent's period-t payoff as $\tilde{u}_t = e_t - \beta\left(\frac{c}{2}e_t^2\right)$. In this model of the agent's payoff, y_t acts as an observable signal of the agent's effort and is of no intrinsic value to the agent (but has instrumental value). Our analysis in the paper remains unaffected under this alternate model of the agent's payoff.

The principal observes the outcome y_t but not the effort exerted by the agent e_t in period-t. The principal's period-t payoff from an outcome y_t and a recommended (or conjectured) effort $\widehat{e}_t$ is:

$$\tilde{v}_t(y_t, \widehat{e}_t) = y_t - \gamma\left(\frac{c}{2}\widehat{e}_t^2\right), \tag{4}$$

where $\gamma \in [0,1)$ denotes the extent to which the principal under-emphasizes the agent's cost of effort. Observe, from (3) and (4), that the principal does not fully internalize the agent's cost of effort. As we will soon show, in equilibrium, the agent finds it optimal to exert the principal's recommended effort. Equivalently, the principal holds rational beliefs about the agent's effort, i.e., in equilibrium, $e_t = \widehat{e}_t$.

Let $\tilde{U}$ and $\tilde{V}$ denote, respectively, the agent's and the principal's sum of the undiscounted payoffs across both periods, i.e., $\tilde{U} = \tilde{u}_1 + \tilde{u}_2$, and $\tilde{V} = \tilde{v}_1 + \tilde{v}_2$. The agent and the principal are risk-neutral and maximize the sum of the undiscounted payoffs across both periods.

3.2. Strategies

The principal's strategy, which we denote by $(\widehat{e}_1, \widehat{\alpha}(\cdot), \widehat{e}_2(\cdot))$, consists of the following:

(a) The principal recommends the agent an effort $\widehat{e}_1 \in \mathbb{R}^+$ in period-1.

(b) The principal commits to a menu of probability mass functions (lotteries), $\widehat{\alpha}(\cdot)$, to provide a unit of the resource to the agent, based on the realized output in period-1. Formally,

$$\widehat{\alpha} : \mathbb{R} \mapsto \Upsilon(\Phi), \tag{5}$$

i.e., $\widehat{\alpha}(\cdot)$ assigns a probability mass function over Φ for any output in period-1. Here, $\Upsilon(\Phi)$ denotes the set of all probability mass functions over Φ. Since $|\Phi| = 2$, $\Upsilon(\Phi)$ corresponds to all Bernoulli distributions. Let $\phi \in \Phi$ denote the realization of the probability mass function.

(c) The principal recommends the agent an effort strategy $\widehat{e}_2(\cdot)$ in period-2, based on the type of resource provided to the agent:

$$\widehat{e}_2 : \Phi \mapsto \mathbb{R}^+.$$

In general, the principal's recommended period-2 effort strategy can depend on the entire observable history thus far. That is, let $h_2 = (y_1, \phi)$ and let $\mathcal{H}_2$ denote the set of all possible period-2 histories. Then, $\widehat{e}_2 : \mathcal{H}_2 \mapsto \mathbb{R}^+$. However, it is straightforward to see that the only relevant part of h_2 is ϕ.

Since, in our case of $\Phi = \{B, S\}$, a probability mass function corresponds to a Bernoulli distribution (a one-parameter distribution), with a mild abuse of notation, let $\widehat{\alpha}(y_1)$ denote the following:

$$\widehat{\alpha}(y_1) = \mathbb{P}\left[\phi = S | y_1\right], \tag{6}$$

We restrict attention to $\widehat{\alpha}(\cdot)$ that are integrable. The tuple $(\widehat{e}_1, \widehat{\alpha}(\cdot), \widehat{e}_2(\cdot))$ denotes a contract. Let $\mathcal{C}$ denote the set of all contracts.

The agent's strategy, which we denote by $(e_1, e_2(\cdot))$, consists of the following:

(a) Effort $e_1 \in \mathbb{R}^+$ in period-1.

(b) Effort strategy $e_2(\cdot)$ based on the type of resource provided in period-2:

$$e_2 : \Phi \mapsto \mathbb{R}^+.$$

The **sequence of events** in the interaction between the principal and agent is as follows:

1. The principal proposes a contract $(\widehat{e}_1, \widehat{\alpha}(\cdot), \widehat{e}_2(\cdot)) \in \mathcal{C}$.
2. The agent chooses effort e_1.
3. Period-1 output y_1 is realized according to (1).
4. The principal provides a unit of the resource of type ϕ to the agent based on the probability mass function (lottery) $\widehat{\alpha}(y_1)$.
5. The agent chooses effort e_2.
6. Period-2 output y_2 is realized according to (2).

3.3. Principal's Problem

Let $u_t = \mathbb{E}[\tilde{u}_t]$ (resp., $v_t = \mathbb{E}[\tilde{v}_t]$) denote the expected payoff to the agent (resp., principal) in period-t and $U = \mathbb{E}[\tilde{U}]$ (resp., $V = \mathbb{E}[\tilde{V}]$) denote the expected sum of payoffs of the agent (resp., principal) across both periods.

The principal's problem is as follows:

$$\max_{(\widehat{e}_1,\widehat{\alpha}(\cdot),\widehat{e}_2(\cdot))\in\mathcal{C}} V(\widehat{e}_1,\widehat{\alpha}(\cdot),\widehat{e}_2(\cdot)) \tag{7}$$

$$\text{s.t.} \quad \widehat{e}_1 \in \arg\max_{e_1\geq 0} U(e_1), \qquad \text{(Agent's Stage 1 Problem)}$$

$$\widehat{e}_2(\phi) \in \arg\max_{e_2\geq 0} u_2(e_2;\phi) \quad \text{for} \quad \phi\in\Phi. \qquad \text{(Agent's Stage 2 Problem)}$$

3.4. Agent's Period-2 Problem

At the start of period-2, the agent is provided access to either the superior resource ($\phi = S$) or the base-quality resource ($\phi = B$). The agent's period-2 problem is as follows:

$$\max_{e_2\in\mathbb{R}^+} \quad \underbrace{\mathbb{E}_{y_2}\left[y_2|\phi,e_2\right]}_{=k_\phi e_2} - \beta\left(\frac{c}{2}e_2^2\right)$$

where $y_2|\phi, e_2$ is shown in (2). The agent's optimal effort in period-2 is:

$$e_2^\star(\phi) = \frac{k_\phi}{\beta c}. \tag{8}$$

Let $u_2^\star(\phi)$ denote the agent's period-2 payoff at their optimal period-2 effort, i.e.,

$$u_2^\star(\phi) = u_2(e_2^\star(\phi);\phi) = \frac{k_\phi^2}{2\beta c}.$$

For convenience, define the agent's *payoff premium* due to the superior resource as follows:

$$\Delta = u_2^\star(S) - u_2^\star(B) = \frac{k_S^2 - k_B^2}{2\beta c}. \tag{9}$$

3.5. Agent's Period-1 Problem

The agent's period-1 problem is as follows:

$$\max_{e_1\in\mathbb{R}^+} U(e_1) = u_1(e_1) + \mathbb{E}_{y_1|e_1}\left[\mathbb{E}_\phi\left[u_2^\star(\phi)\middle|y_1\right]\right]$$

Using (1) and (9), the r.h.s. above simplifies as follows:

$$\begin{aligned} U(e_1) &= \left(e_1 - \beta\frac{c}{2}e_1^2\right) + \mathbb{E}_{y_1|e_1}\left[\widehat{\alpha}(y_1)u_2^\star(S) + (1-\widehat{\alpha}(y_1))u_2^\star(B)\right] \\ &= \left(e_1 - \beta\frac{c}{2}e_1^2\right) + \Delta\underbrace{\int_{y_1\in\mathbb{R}} \widehat{\alpha}(y_1)\psi\left(y_1\middle|e_1\right)dy_1}_{\mathbb{P}[\phi=S|e_1]} + \frac{k_B^2}{2\beta c} \end{aligned} \tag{10}$$

In the r.h.s. above, the last term is a constant and independent of e_1. In the second term, the integral denotes the probability of receiving access to the superior resource if the agent exerts effort e_1 in period-1.

3.6. Restating the Principal's Problem

We restate the principal's problem using the solutions to the agent's period-2 and period-1 problems. From the agent's period-2 problem, it follows that the principal's recommended period-2 strategy must be incentive compatible to the agent, i.e., $\widehat{e}_2(\cdot) = e_2(\cdot)$ as shown in (8). We simplify the principal's payoff under a recommended period-1 effort $\widehat{e}_1$, menu of probability mass functions $\widehat{\alpha}(\cdot)$, and the recommended period-2 strategy $\widehat{e}_2(\cdot)$ using (8) as follows:

$$
\begin{aligned}
V(\widehat{e}_1, \widehat{\alpha}(\cdot)) &= v_1(\widehat{e}_1) + \mathbb{E}_{y_1|\widehat{e}_1}\left[\mathbb{E}_{\phi}\left[v_2^{\star}(\phi)\middle| y_1\right]\right] \\
&= \left(\widehat{e}_1 - \gamma\frac{c}{2}\widehat{e}_1^2\right) + \mathbb{E}_{y_1|\widehat{e}_1}\left[\widehat{\alpha}(y_1)v_2^{\star}(S) + (1-\widehat{\alpha}(y_1))\, v_2^{\star}(B)\right] \\
&= \left(\widehat{e}_1 - \gamma\frac{c}{2}\widehat{e}_1^2\right) + \Delta\left(2-\frac{\gamma}{\beta}\right)\underbrace{\int_{y\in\mathbb{R}} \widehat{\alpha}(y_1)\psi(y_1|\widehat{e}_1)dy_1}_{\mathbb{P}[\phi=S|e_1]} + \frac{k_B^2}{2\beta c}\left(2-\frac{\gamma}{\beta}\right).
\end{aligned} \tag{11}
$$

The last term in the r.h.s. above is a constant and independent of $\widehat{e}_1, \widehat{\alpha}(\cdot)$. By ignoring the constant terms in (10) and (11), the principal's problem in (7) can be written as the following bilevel optimization problem, denoted by Problem MH (for <u>m</u>oral <u>h</u>azard):

$$
\left.
\begin{aligned}
&\max_{\widehat{e}_1, \widehat{\alpha}(\cdot)} \left(\widehat{e}_1 - \gamma\frac{c}{2}\widehat{e}_1^2\right) + \left(2-\frac{\gamma}{\beta}\right)\Delta\int_{y_1\in\mathbb{R}} \widehat{\alpha}(y_1)\psi\left(y_1\middle|\widehat{e}_1\right)dy_1 \\
&\text{s.t. } \widehat{e}_1 \in \arg\max_{e_1\in\mathbb{R}^+} \left(e_1 - \beta\frac{c}{2}e_1^2\right) + \Delta\int_{y_1\in\mathbb{R}} \widehat{\alpha}(y_1)\psi\left(y_1\middle|e_1\right)dy_1
\end{aligned}
\right\} \quad (\text{Problem MH})
$$

We say that a contract "induces" a period-1 effort e_1 if e_1 maximizes the agent's payoff under the contract.

Before we analyze Problem MH, we make an important observation. Note that the principal does not directly benefit from holding/owning the resource. Furthermore, the agent strictly benefits from receiving the superior resource (relative to the base-quality resource). The agent's output in period-2 is stochastically higher if the agent receives the superior resource than if they receive the base quality resource, and the principal's payoff is strictly increasing in the period-2 output. Despite these incentives, we will show why the principal benefits in throttling access to the superior resource.

4. A Benchmark: The Free-Access Contract

Recall that our focus in Sections 3-7 is on settings where the principal has a sufficient quantity, if needed, of the superior resource to support the entire population of beneficiaries. An important benchmark to compare the optimal contract (i.e., solution to Problem MH, which we formulated in Section 3.6) is the *free-access* contract, where the principal *always* provides the agent with the superior resource. That is, under the free-access contract, we have:

$$\widehat{\alpha}(y_1) = 1 \text{ for all } y_1 \in \mathbb{R}.$$

We denote the free-access contract by FREE. For convenience, let e_A denote the agent's preferred period-1 effort, i.e., the effort that maximizes u_1:

$$e_A = \frac{1}{\beta c}. \tag{12}$$

The following result presents the optimal effort of the agent in period-1 under FREE.

THEOREM 1 **(Free Access Contract)**. *Under* FREE, *the agent's optimal period-1 effort is:*

$$e_1^\star = e_A.$$

Observe that under FREE, the agent chooses their preferred period-1 effort, and hence maximizes their period-1 payoff. Besides, the agent always receives the superior resource, thereby maximizing their period-2 payoff. Thus, the free-access contract maximizes the agent's payoff across both periods.

Further, observe that FREE maximizes the principal's period-2 payoff, but not their period-1 payoff. The principal's preferred period-1 effort, i.e., the effort that maximizes the principal's period-1 payoff, denoted by e_P, is:

$$e_P = \frac{1}{\gamma c}.$$

The principal's preferred period-1 effort is strictly larger than the agent's preferred period-1 effort (i.e., $e_P > e_A$) since the principal does not fully internalize the agent's cost of effort and the agent overestimates their cost of effort ($\gamma < \beta$). A real-world example of an NPO that, to our knowledge, has adopted the free-access contract is Samarthanam (`https://www.samarthanam.org/livelihood-resource-center/`), which works exclusively for educating disabled persons.

5. Analysis of PROBLEM MH

We now solve for an optimal contract, i.e., identify an optimal solution to PROBLEM MH. Recall that a contract "induces" a period-1 effort e_1 from the agent if e_1 maximizes the agent's payoff under that contract. To develop ideas in a systematic manner, we proceed as follows:

1. We first identify the set of all inducible efforts in period-1 (Theorem 2).
2. Consider an inducible effort, say e_1. Among all contracts that induce e_1, we identify an optimal contract (for the principal) that induces e_1 (Theorem 3).
3. We identify the optimal period-1 effort that the principal chooses to induce (Theorem 4).

Define the following efforts:

$$\underline{e} = \max\left\{ e_A\left(1 - \frac{\Delta}{\sqrt{2\pi}\sigma}\right), 0\right\} \text{ and } \overline{e} = e_A\left(1 + \frac{\Delta}{\sqrt{2\pi}\sigma}\right). \tag{13}$$

We make the following regularity assumption, which states that the agent's period-1 output is sufficiently noisy. As we will soon demonstrate, Assumption 1 makes our analysis tractable.

Assumption 1 **(σ is "Sufficiently" Large).** $\sigma > \underline{\sigma} \triangleq \sqrt{\frac{\Delta}{\beta c\sqrt{2\pi e}}}$.

The following result characterizes the set of inducible efforts in period-1.

Theorem 2 **(Inducible Period-1 Efforts).** *The set of all inducible efforts in period-1 is:* $\mathcal{E} = [\underline{e}, \overline{e}]$. *Stated differently, there exists a contract that induces e_1 (i.e., e_1 maximizes the agent's payoff under that contract) if and only if $e_1 \in \mathcal{E}$.*

Theorem 2 shows that the principal can induce a period-1 effort e_1 from the agent iff $e_1 \in \mathcal{E}$. Recall that Free induces a period-1 effort $e_A = \frac{1}{\beta c}$. From the principal's standpoint, any contract that induces a period-1 effort less than e_A is dominated by Free. Therefore, the equilibrium period-1 effort induced by the principal is *at least* e_A.

Next, we define a special class of contracts called "threshold" contracts.

5.1. Threshold Contracts

Fix $\tau \in \mathbb{R}$. Consider a contract where the principal's resource-allocation strategy $\widehat{\alpha}(\cdot)$ is as follows:

$$\widehat{\alpha}(y_1) = \begin{cases} 0, \text{ if } y_1 \le \tau; \\ 1, \text{ if } y_1 > \tau. \end{cases} \tag{14}$$

Then, the agent's problem is as follows:

$$\begin{aligned} \max_{e_1}\ U(e_1) &= u_1(e_1) + \Delta \int_{y_1=\tau}^{\infty} \psi(y_1 \big| e_1) dy_1 \\ &= \left(e_1 - \beta\frac{c}{2}e_1^2\right) + \Delta\left(1 - \Psi\left(\tau\big|e_1\right)\right). \end{aligned}$$

Since the r.h.s. is smooth, first order conditions (f.o.c.'s) must hold at optimality. Therefore,

$$U'(e_1) = 0 \implies (1 - \beta c e_1) + \Delta\psi(\tau|e_1) = 0.$$

The result below establishes the agent's best-response to the above $\widehat{\alpha}(\cdot)$.

Lemma 1 **(Agent Best-Response to (14)).** *Consider $\widehat{\alpha}(\cdot)$ as shown in (14). The agent's period-1 best-response is unique and solves the following equation:*

$$e_1 = e_A\left(1 + \Delta\psi\left(\tau\big|e_1\right)\right) \tag{15}$$

The agent's best-response in (15) is increasing in τ if $\tau \le \overline{e}$ and is decreasing in τ if $\tau > \overline{e}$, where $\overline{e}$ is as defined in (13). Further, at $\tau = \overline{e}$, the agent's best-response $e_1 = \overline{e}$.

The fixed-point equation (15) follows from the first-order condition of the agent's payoff. Assumption 1 ensures that the agent's best-response – namely, the fixed point in (15) – is unique. We demonstrate this in Figure 2.

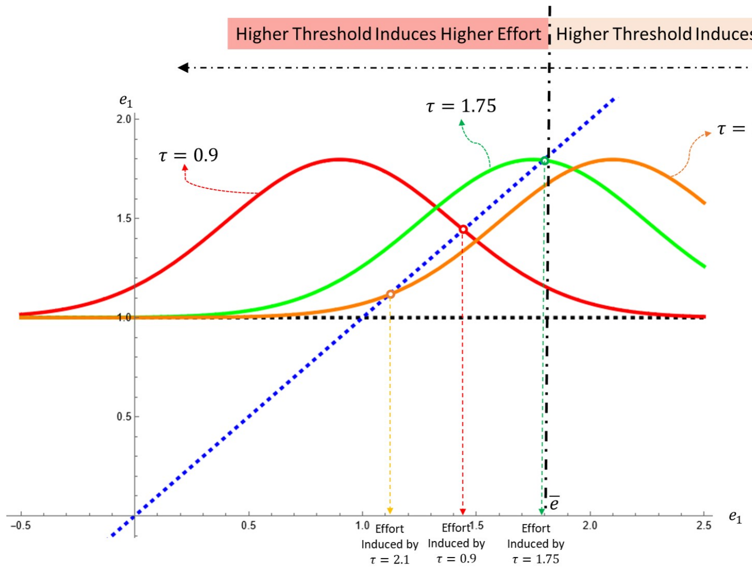

Figure 2 **Illustrating the solution to the fixed-point equation (15). Both the X-axis and Y-axis correspond to e_1. The parameter values for this figure are $k_S = 1, k_B = 0$, $\beta = 1$, $c = 1$, $\sigma = 0.5$. $e_A = 1$, $\Delta = 1$, $\underline{\sigma} \approx 0.491$. For these values, we have $\bar{e} \approx 1.79$. Observe that e_1 is increasing in τ if $\tau < \bar{e} \approx 1.79$ and decreasing in τ if $\tau > \bar{e}$. The red curve corresponds to the r.h.s. of (15) at $\tau = 0.9$, the green (resp., orange) curve corresponds to the r.h.s. of (15) at $\tau = 1.75$ (resp., $\tau = 2.1$). The solutions to the fixed-point equation (i.e., induced first-period agent effort) are shown by dotted circles.**

We now formally define a threshold contract.

Definition 1 (Threshold Contract). A threshold contract, with threshold $\tau \in \mathbb{R}$, is the tuple $(\widehat{e}_1, \widehat{\alpha}(\cdot), \widehat{e}_2(\cdot))$, where:

(a) $\widehat{e}_1$ is the solution to (15),

(b) $\widehat{\alpha}(\cdot)$ is as shown in (14), and

(c) $\widehat{e}_2(\cdot)$ is as shown in (8).

Since a threshold contract is fully specified by a single parameter τ, we will avoid referring to a threshold contract using the tuple above, and simply refer to it as the threshold contract with the threshold τ. Let $\mathcal{C}^{\text{THR}}$ denote the set of threshold contracts. Observe that $\text{Free} \in \mathcal{C}^{\text{THR}}$, where the threshold $\tau = -\infty$.

The result below characterizes the set of period-1 efforts that can be induced by threshold contracts.

Lemma 2 **(Inducible Period-1 Efforts by Threshold Contracts)**. *The set of inducible efforts in period-1 by threshold contracts, denoted by $\mathcal{E}^{\text{THR}}$, is:*

$$\mathcal{E}^{\text{THR}} = [e_A, \bar{e}] .$$

Further, consider any $e_1 \in \mathcal{E}^{\text{THR}}$. Let $\tau_{(-)}$ and $\tau_{(+)}$ be defined as follows:

$$\tau_{(-)} = e_1 + \psi^{-1}_{(-)}\left(\frac{1}{\Delta}\left(\frac{e_1 - e_A}{e_A}\right)\Big|e_1\right) \text{ and } \tag{16}$$

$$\tau_{(+)} = e_1 + \psi^{-1}_{(+)}\left(\frac{1}{\Delta}\left(\frac{e_1 - e_A}{e_A}\right)\Big|e_1\right). \tag{17}$$

There are exactly two threshold contracts that induce e_1: a threshold contract with threshold $\tau_{(-)}$ and one with threshold $\tau_{(+)}$. Here, $\tau_{(-)} \leq \tau_{(+)}$, where the inequality is strict if $e_1 < \bar{e}$.

The first part of Lemma 2 follows from the second part of Lemma 1. The second part of Lemma 2 shows that there are two threshold contracts – with thresholds $\tau_{(-)}$ and $\tau_{(+)}$ – that induce a given effort. Recall from (15) and Lemma 1 that the agent's best response increases and then decreases. After rearranging (15), we have:

$$\psi\left(\tau\Big|e_1\right) = \frac{1}{\Delta}\left(\frac{e_1 - e_A}{e_A}\right).$$

For a fixed $e_1 \in \mathcal{E}^{\text{THR}}$, the equation above identifies the p.d.f. value associated with τ. While both these threshold contracts induce e_1, we will show why the principal strictly prefers one of them over the other. To do so, we first define the probability that an agent receives the superior resource under a threshold contract with threshold τ: $\mathbb{P}[\Phi = S]\Big|_{\tau} = 1 - \Psi\left(\tau\Big|e_1\right)$, where e_1 solves (15). Now, consider an induced period-1 effort $e_1 \in \mathcal{E}^{\text{THR}}$, and the two thresholds $\tau_{(-)}$ and $\tau_{(+)}$ that induce e_1 (as shown above in Lemma 2). Observe that the r.h.s. – the probability that the agent receives the superior resource – is larger under $\tau_{(-)}$ than under $\tau_{(+)}$. The principal's payoff in period-1 is identical under the two threshold contracts, since both contracts induce e_1. However, recall that the principal's period-2 payoff is increasing in the probability that the agent receives the superior resource. Since $\tau_{(-)}$ leads to a higher probability of the agent receiving the superior resource, we have the following result.

Lemma 3 **(Preference Between the Two Threshold Contracts)**. *Consider an arbitrary $e_1 \in \mathcal{E}^{\text{THR}}$. Suppose the principal chooses to induce a period-1 effort e_1 using a threshold contract. Then, the principal prefers a threshold contract with threshold $\tau_{(-)}$ over that with threshold $\tau_{(+)}$, where the preference is strict if $e_1 < \bar{e}$. That is, $V\left(\tau_{(-)}\right) \geq V\left(\tau_{(+)}\right)$ where the inequality is strict if $e_1 < \bar{e}$.*

Note that $\tau_{(-)} \leq \bar{e}$; see (A.28). Thus, a consequence of Lemma 3 is that threshold contracts with thresholds larger than $\bar{e}$ cannot arise in equilibrium, since they are dominated. Henceforth, whenever we refer to a threshold contract that induces e_1, we refer to the corresponding threshold contract with threshold $\tau_{(-)}$. However, it is yet to be determined if the principal chooses a threshold contract in equilibrium. In other words, we are yet to demonstrate that an optimal solution to Problem MH is a threshold contract. In what follows, we show the following *stronger result*.

THEOREM 3 **(Sufficiency of Threshold Contracts)**. *Fix $e_1 \in \mathcal{E}$, $e_1 \geq e_A$. Suppose the principal chooses to induce a period-1 effort e_1 from the agent. Then, among the set of all contracts that induce e_1, the threshold contract that induces e_1 is optimal for the principal.*

Together with Lemmas 2 and 3, a consequence of Theorem 3 is that it suffices to restrict attention to the set of threshold contracts $\mathcal{C}^{\text{THR}}$ with thresholds $\tau \leq \overline{e}$.

In the next subsection, we restrict attention to threshold contracts and identify an optimal solution to PROBLEM MH, i.e., an optimal contract for the principal.

5.2. An Optimal Contract

In our search for an optimal contract, from the earlier analysis, it suffices to consider threshold contracts with thresholds $\tau \leq \overline{e}$. Further, from (16) in Lemma 1, observe that there is a one-to-one correspondence between the threshold and the induced effort. Consequently, we identify the period-1 effort that the principal chooses to induce. The corresponding threshold can be identified using (16).

Let $\widehat{e}_1$ denote the induced period-1 effort, and $\tau(\widehat{e}_1)$ denote the corresponding threshold (identified from (16)). From Lemma 2, the threshold $\tau(\widehat{e}_1)$ is increasing in $\widehat{e}_1$, and $\tau(\widehat{e}_1) \leq \widehat{e}_1$ where the inequality is strict if $e_1 < \overline{e}$. We rewrite the principal's problem as follows:

$$\max_{\widehat{e}_1 \in \mathcal{E}^{\text{THR}}} V(\widehat{e}_1) = \overbrace{\left(\widehat{e}_1 - \gamma \frac{c}{2} \widehat{e}_1^2\right)}^{v_1(\widehat{e}_1)} + \overbrace{\Delta \left(2 - \frac{\gamma}{\beta}\right) \left(\underbrace{1 - \Psi\left(\tau(\widehat{e}_1) \middle| \widehat{e}_1\right)}_{\mathbb{P}[\phi = S]\big|_{\widehat{e}_1}} \right)}^{v_2(\widehat{e}_1)} \tag{18}$$

The result below presents some useful properties of $V(\widehat{e}_1)$.

LEMMA 4 **(Properties of $V(\widehat{e}_1)$)**. *The following two statements hold.*

(a) $\tau(\widehat{e}_1) - \widehat{e}_1$ *is increasing in* $\widehat{e}_1 \in \mathcal{E}^{\text{THR}}$.

(b) $V(\widehat{e}_1)$ *is strictly concave in* $\widehat{e}_1$.

A consequence of Lemma 4(a) is that the probability of the agent receiving the superior resource, $1 - \Psi\left(\tau(\widehat{e}_1) \middle| \widehat{e}_1\right)$, is decreasing in $\widehat{e}_1$. That is, the principal induces a higher effort by "throttling" access to the superior resource. In Lemma 4(b), it is trivial that $v_1(\widehat{e}_1)$ is strictly concave in $\widehat{e}_1$. The non-trivial part is to show the concavity of the probability of the agent receiving the superior resource in the induced effort, i.e., $1 - \Psi\left(\tau(\widehat{e}_1) \middle| \widehat{e}_1\right)$ is concave in $\widehat{e}_1$. From Lemma 4(b), if the optimal solution is in the interior of $\mathcal{E}^{\text{THR}}$, then the first-order condition is necessary and sufficient. Using Leibnitz rule, we have:

$$V'(\widehat{e}_1) = \overbrace{\left(\underbrace{1-\gamma c \widehat{e}_1}_{=1-\frac{\widehat{e}_1}{e_P}}\right)}^{\text{marginal period-1 payoff, } v_1'(\widehat{e}_1)} + \overbrace{\left(\Delta\left(2-\frac{\gamma}{\beta}\right)\underbrace{\left(-\psi\left(\tau(\widehat{e}_1)\middle|\widehat{e}_1\right)\right)(\tau'(\widehat{e}_1)-1)}_{=\frac{d}{d\widehat{e}_1}\left(\mathbb{P}[\phi=S]\Big|_{\widehat{e}_1}\right)}\right)}^{\text{marginal period-2 payoff, } v_2'(\widehat{e}_1)}.$$

In the r.h.s. above, the first term represents the principal's marginal period-1 payoff from an effort $\widehat{e}_1$ by the agent. Recall that $e_A < e_P$. Therefore, the principal benefits if the agent exerts a period-1 effort larger than e_A. To induce a higher effort, however, requires the principal to increase the threshold, thereby throttling access to the superior resource (see Lemma 4(a)). This hurts the principal's period-2 payoff, since the principal's period-2 payoff is increasing in the probability that the agent receives the superior resource. This tradeoff represents the inherent tension in the principal's problem. At optimality, the induced effort balances the principal's gain in the period-1 payoff and their loss in the period-2 payoff. Further simplification yields the following:

$$V'(\widehat{e}_1) = \left(1-\frac{\widehat{e}_1}{e_P}\right) - \frac{\left(2-\frac{\gamma}{\beta}\right)\sigma}{e_A\sqrt{2\log\left(\frac{\overline{e}-e_A}{\widehat{e}_1-e_A}\right)}}$$

The r.h.s. in the above equation follows from (16) and from algebraic manipulation. Using the expression in the r.h.s. above, at the extremes, we have

$$V'(e_A) = 1-\frac{e_A}{e_P} \quad \text{and} \quad V'\left(\min\{\overline{e}, e_P\}\right) = \begin{cases} -\infty, & \text{if } \overline{e} \le e_P; \\ -\frac{\left(2-\frac{\gamma}{\beta}\right)\sigma}{e_A\sqrt{2\log\left(\frac{\overline{e}-e_A}{e_P-e_A}\right)}}, & \text{if } \overline{e} > e_P. \end{cases}$$

Observe that $V'(e_A) > 0$ (since $e_A < e_P$), and $V'(\min\{\overline{e}, e_P\}) < 0$. Consequently, the optimal solution is in the interior of $[e_A, \min\{\overline{e}, e_P\}]$. Since $V(\cdot)$ is strictly concave and $V'(e_A) > 0 > V'(\min\{\overline{e}, e_P\})$, the first-order condition is necessary and sufficient. Using the first-order condition, we have

$$V'(\widehat{e}_1) = 0 \implies 1-\frac{\widehat{e}_1}{e_P} = \frac{\left(2-\frac{\gamma}{\beta}\right)\sigma}{e_A\sqrt{2\log\left(\frac{\overline{e}-e_A}{\widehat{e}_1-e_A}\right)}}. \tag{19}$$

After rearranging the above equation, we have the following result.

THEOREM 4 **(Equilibrium Period-1 Effort)**. *In equilibrium, the period-1 effort that the principal chooses to induce from the agent is the unique solution to the following fixed-point equation:*

$$\widehat{e}_1 = e_A + (\overline{e}-e_A)\exp\left(-\frac{1}{2}\left(\frac{\left(2-\frac{\gamma}{\beta}\right)\frac{\sigma}{e_A}}{1-\frac{\widehat{e}_1}{e_P}}\right)^2\right). \tag{20}$$

Algebraically, observe that the r.h.s. in (20) is strictly decreasing in $\widehat{e}_1$, and the l.h.s. is smaller (resp., larger) than the r.h.s. at $\widehat{e}_1 = e_A$ (resp., $\widehat{e}_1 = \min\{\overline{e}, e_P\}$). Consequently, the above fixed-point equation provides a unique solution in the interior of $[e_A, \min\{\overline{e}, e_P\}]$. The corresponding threshold that the principal chooses can be expressed in terms of the induced effort using (16), as follows:

$$\tau = \widehat{e}_1 + \psi_{(-)}^{-1}\left(\frac{1}{\Delta}\left(\frac{\widehat{e}_1 - e_A}{e_A}\right)\Big|\widehat{e}_1\right) \tag{21}$$

The probability that the agent receives the superior resource in equilibrium can be expressed as follows:

$$\mathbb{P}\left[\phi = S\right]\Big|_{\widehat{e}_1} = 1 - \Psi\left(\tau(\widehat{e}_1)\Big|\widehat{e}_1\right) = 1 - \Psi_0\left(\psi_{0(-)}^{-1}\left(\frac{\sigma}{\Delta}\left(\frac{\widehat{e}_1 - e_A}{e_A}\right)\right)\right). \tag{22}$$

To summarize our analysis thus far, we have established the sufficiency of restricting attention to the class of threshold contracts (Section 5.1) and have identified an optimal contract among threshold contracts (Section 5.2). An optimal contract for the principal – i.e., an optimal solution to Problem MH – is a threshold contract where the threshold is given by (21).

6. Comparative Statics

In this section, we analyze how the optimal contract identified in Section 5 changes with the parameters of the environment. Specifically, we provide comparative statics of two key quantities of interest – the equilibrium effort, and the probability that the agent receives the superior resource in equilibrium – with respect to the model parameters, namely the cost of effort, the noise in the period-1 outcome, the efficacy of the superior resource, and the behavioral/incentive misalignment parameters.

6.1. Induced Effort

Recall that (20) provides a fixed-point equation that identifies the period-1 effort that the principal chooses to induce in equilibrium from the agent. In the result below, we show how the induced effort changes based on the parameters of the environment.

Theorem 5. *The agent's equilibrium period-1 effort $\widehat{e}_1$ is*

(a) ***(Noise)*** *decreasing in σ,*
(b) ***(Cost of Effort)*** *decreasing in c,*
(c) ***(Efficacy Premium of the Superior Resource)*** *increasing in k_S and decreasing in k_B,*
(d) ***(Incentive Misalignment Parameters)*** *decreasing in γ and β.*

Part (a) shows that the principal induces a lower effort when the environment is more noisy. In a more noisy environment, the agent's period-1 output is less informative of their effort, and hence the agent has a lesser incentive to exert effort. Part (b) shows that the induced effort is lower as exerting effort becomes more costly. Part (c) shows the role of the efficacy of the superior resource. A higher efficacy premium leads to a greater effort. Part (d) shows the role of misalignment of incentives. If γ and β are close to 1, then the incentives are more closely aligned. Notice that if the principal internalizes the agent's disutility from effort to a greater extent, the induced effort is lower.

6.2. Extent of Throttling

The extent of throttling – the probability that the agent receives the superior resource – denoted by $\mathbb{P}[\phi = S]\Big|_{\widehat{e}_1}$, is shown in (22). Observe that this probability is decreasing in the quantity $\frac{\sigma}{\Delta}\frac{\widehat{e}_1 - e_A}{e_A}$, which is the term in the parenthesis in the r.h.s. of (22).

THEOREM 6. *(a)* ***(Noise and Cost of Effort)*** *Suppose the following holds:*

$$c\sigma > \frac{\gamma}{\beta(\beta-\gamma)}\frac{k_S^2 - k_B^2}{\sqrt{2\pi}}.$$

Then, $\mathbb{P}[\phi = S]\Big|_{\widehat{e}_1}$ *is increasing in c and increasing in σ.*

(b) ***(Efficacy Premium of the Superior Resource)*** $\mathbb{P}[\phi = S]\Big|_{\widehat{e}_1}$ *is increasing in k_S and decreasing in k_B.*

(c) ***(Incentive Misalignment Parameters)*** $\mathbb{P}[\phi = S]\Big|_{\widehat{e}_1}$ *is increasing in γ and decreasing in β.*

Observe that the probability of receiving the superior resource is increasing in the cost of effort (c) and in the amount of noise (σ) in the environment. This is especially surprising, considering our observation in Theorem 5(a) and (b) that effort is decreasing in the cost of effort and the noise in the environment. The underlying reasoning is as follows. First, for any level of stage-2 effort, the agent's output is higher with the superior resource, relative to that with the base-quality resource. Thus, all else being equal, the principal prefers to provide the superior resource to the agent in period 2. Consider the effect of σ: An increase in σ makes the agent's period-1 output less informative of the agent's effort, and hence the agent has a lesser incentive to exert effort. It is convenient to consider the extreme case: In the limit (as $\sigma \to \infty$), the period-1 output is uninformative of the agent's effort. Thus, the principal is unable to affect the agent's period-1 effort, and hence provides the superior resource with probability 1 in period-2 (i.e., $\tau = -\infty$). While τ and $\widehat{e}_1$ both decrease in σ, the term $\tau - \widehat{e}_1$ is decreasing in σ (i.e., τ decreases at a faster rate than $\widehat{e}_1$). Consequently, the probability of the agent receiving the superior resource is increasing in σ. A similar reasoning applies to the effect of c.

7. Extension: Multiple Types of Resources

Recall our base model in Section 3, where we assume that the principal has access to two types of resources. We now extend the base model to a setting where the principal has access to a finite set of two or more types of resources; i.e., $|\Phi| \geq 2$ but finite. Our main result in this section is that the analysis under multiple types of resources can be reduced to the one under two types of resources. Consequently, it is "without loss of generality" to restrict attention to the case of two types of resources.

Let k_ϕ denote the efficacy of resource $\phi \in \Phi$. We assume that for any $\phi, \phi' \in \Phi$, $k_\phi \neq k_{\phi'}$, i.e., no two resources have identical efficacy. Denote the following:

$$\phi \succ \phi' \Leftrightarrow k_\phi > k_{\phi'}.$$

Thus, $(\Phi, \succ)$ is an ordered set. Let $\underline{\phi}$ (resp., $\overline{\phi}$) denote the unique minimal (resp., maximal) element of Φ, i.e., the resource with the least (resp., greatest) efficacy:

$$\underline{\phi} = \arg\min_{\phi\in\Phi} k_\phi \text{ and } \overline{\phi} = \arg\max_{\phi\in\Phi} k_\phi.$$

The principal's and the agent's payoffs are as shown in Section 3.1, and their strategies are as shown in Section 3.2. As defined in (5), the principal's resource-allocation strategy is a menu of probability mass functions (i.e., lotteries), $\widehat{\alpha}(\cdot)$, over the set Φ to provide a unit of the resource to the agent, based on the realized output in period-1. With a mild abuse of notation, we denote

$$\widehat{\alpha}(y_1) \equiv \sum_{\phi\in\Phi} \widehat{\alpha}(\phi|y_1) \circ \phi.$$

That is, $\widehat{\alpha}(\phi|\cdot)$ corresponds to the probability (mass) with which the agent receives the resource $\phi \in \Phi$ under the strategy $\widehat{\alpha}(\cdot)$.

Analogous to the earlier analysis, we can write the principal's problem in this setting, denoted by Problem MH − Multiple as follows.

$$\left.\begin{aligned} &\max_{\widehat{e}_1, \widehat{\alpha}(\cdot)} \left(\widehat{e}_1 - \gamma\frac{c}{2}\widehat{e}_1^2\right) + \left(2 - \frac{\gamma}{\beta}\right) \sum_{\phi\in\Phi} \frac{k_\phi^2}{2\beta c} \int_{y_1\in\mathbb{R}} \widehat{\alpha}\left(\phi \middle| y_1\right) \psi\left(y_1 \middle| \widehat{e}_1\right) dy_1 \\ &\text{s.t. } \widehat{e}_1 \in \arg\max_{e_1\in\mathbb{R}^+} \left(e_1 - \beta\frac{c}{2}e_1^2\right) + \sum_{\phi\in\Phi} \frac{k_\phi^2}{2\beta c} \int_{y_1\in\mathbb{R}} \widehat{\alpha}\left(\phi \middle| y_1\right) \psi\left(y_1 \middle| e_1\right) dy_1 \end{aligned}\right\} \quad (\text{Problem MH} - \text{Multiple})$$

The term $\int_{y_1\in\mathbb{R}} \widehat{\alpha}\left(\phi \middle| y_1\right) \psi\left(y_1 \middle| \widehat{e}_1\right) dy_1$ denotes the probability that the agent receives the resource of type ϕ under the principal's strategy $\widehat{\alpha}(\cdot)$.

We say that a resource $\phi' \in \Phi$ is "irrelevant" for a menu of probability mass functions, $\widehat{\alpha}(\cdot)$, if $\widehat{\alpha}(\phi'|y) = 0$ for all $y \in \mathbb{R}$. A set of resources $\Phi' \subset \Phi$ is irrelevant for $\widehat{\alpha}(\cdot)$ if every resource $\phi' \in \Phi'$ is irrelevant for $\widehat{\alpha}(\cdot)$. In words, a resource ϕ' (resp., a set of resources Φ') is irrelevant if the principal does not provide the resource ϕ' (resp., a resource from the set Φ') under any contingency. We have the following fundamental result.

Theorem 7 **(Irrelevance of Non-Extremal Resources)**. *In equilibrium, the set $\Phi \setminus \{\underline{\phi}, \overline{\phi}\}$ is irrelevant.*

Theorem 7 shows that under multiple types of resources, it suffices to restrict attention to probability mass functions (lotteries) over the extremal resources. Consequently, an optimal solution to Problem MH, where the set of resources is $\{\underline{\phi}, \overline{\phi}\}$, is also optimal for Problem MH – Multiple, where the set of resources is a superset of $\{\underline{\phi}, \overline{\phi}\}$. Recall from the analysis in Section 5 that under two types of resources, in equilibrium, the principal uses a threshold strategy (Theorem 3). Therefore, a threshold strategy over the extremal resources is an optimal solution to Problem MH – Multiple.

Theorem 7 has an immediate implication for the design of resource types: *when the principal has the ability to design multiple types of resources that can be ordered in their efficacies, then it suffices to design a menu with only the two resources that have extremal efficacies.*

8. Impact of Competition and Resource Scarcity

Our base model in Section 3 and analysis thus far assumes that the NPO (principal) has a sufficient amount of superior resources to support all beneficiaries (agents), if needed. Our intent in making this assumption was to uncover the strategic role of throttling access the superior resources to incentivize the agents to exert a higher effort. Nonetheless, in practice, NPOs may have fewer resources than the population of beneficiaries they would ideally like to support. Indeed, the size of the population of beneficiaries is often a strategic choice for an NPO.

In this section, we focus on the case where an NPO has access to fewer superior resources than the size of the population of beneficiaries. We comment on two important questions: (i) How does the competition among agents for scarce resources affect a principal's resource-allocation strategy? (ii) How should an NPO strategically choose the size of the beneficiary population? We denote the number of superior resources available to the principal by m, and the number of agents by n. Let $[n] = \{1, 2, \ldots, n\}$. We assume that $m \leq n$; if $m \geq n$, then the analysis is identical to the case of $m = n$, which corresponds to our base model in Section 3.

8.1. Competition among Agents

To demonstrate the role of competition among the agents, we consider the case of one superior resource and n agents ($m = 1$ and $n \geq 1$). Let y_{j1} denote the period-1 outcome of the agent j, $j \in [n]$, and let $\boldsymbol{y}_1 = (y_{j1})_{j\in[n]}$. The principal's resource-allocation strategy is as follows: Let $\widehat{\alpha}(\boldsymbol{y}_1) = (\widehat{\alpha}_j(\boldsymbol{y}_1))_{j\in[n]}$ where $\widehat{\alpha}_j(\boldsymbol{y})$ denotes the probability that agent j receives the superior resource corresponding to period-1 output $\boldsymbol{y}$. We require that $\widehat{\alpha}(\cdot)$ belong to the n-dimensional standard (probability) simplex. Let $\mathsf{rank}(j)$ denote the rank of agent j in the decreasing order of the agents' period-1 output, where $\mathsf{rank} = 1$ (resp., $\mathsf{rank} = n$) corresponds to the index of the agent with the highest (resp., lowest) period-1 output (with ties broken arbitrarily). We restrict attention to threshold strategies for the principal. That is, for any threshold $\tau \in \mathbb{R}$ that the principal chooses:

$$\widehat{\alpha}_j(\boldsymbol{y}_1) = \begin{cases} 1, \text{ if } \mathsf{rank}(j) = 1 \text{ and } y_{j1} \geq \tau; \\ 0, \text{ o/w.} \end{cases}$$

In words, the principal gives the superior resource to the agent, if any, with the highest period-1 output that exceeds the threshold τ. We restrict attention to the symmetric subgame equilibria among the agents. Consider a choice τ for the principal. Suppose agent j chooses e_{j1}, while all other agents choose e_1. The probability that $\mathsf{rank}(j) = 1$ is as follows:

$$\mathbb{P}\left[\text{agent } j \text{ receives the resource}\middle| e_{j1}, e_1\right] = \int_{y_{j1}=\tau}^{\infty} \psi\left(y_{j1}\middle| e_{j1}\right)\left(\Psi\left(y_{j1}\middle| e_1\right)\right)^{n-1} dy_{j1}.$$

The symmetric (subgame equilibrium) effort induced by the principal e_1 satisfies the following: It is optimal for agent j to choose $e_{j1} = e_1$ if the principal chooses τ and all other agents choose e_1. Formally,

$$e_1 \in \arg\max_{e_{j1} \geq 0} \left\{ u_1(e_{j1}) + \Delta\, \mathbb{P}\left[\text{agent } j \text{ receives the resource}\middle| e_{j1}, e_1\right]\right\}. \tag{23}$$

It is straightforward to verify that the symmetric subgame equilibrium, i.e., the solution to (23) exists and is unique. The principal's problem simplifies to the following:

$$\begin{aligned} \max_{\tau \in \mathbb{R}} V &= n v_1(e_1) + \Delta\left(2 - \frac{\gamma}{\beta}\right)(1 - s^n) \\ \text{where } s &= \mathbb{P}\left[y_{j1} \leq \tau \middle| e_1\right] = \Psi\left(\tau \middle| e_1\right) \text{ and } e_1 \text{ solves (23).} \end{aligned}$$

However, solving the problem above is challenging; hence, we solve the problem numerically. We present the outcome based on our numerical experiments in Figure 3 below.

Insight: Optimal Extent of Competition

Observe from Figure 3 that the induced effort is non-monotone in the extent of competition among the agents. At low levels of competition (low n), the induced effort is increasing in the extent of competition. However, at high levels of competition (high n), the induced effort is decreasing in the extent of competition. This observation has an important implication: *For the NPO, the ideal level of competition among the agents is neither too low nor too high.* Further, note that *the ideal level of competition is (a) increasing in the efficacy of the superior resource (Δ) and (b) decreasing in the noise in the period-1 outcome (σ).*

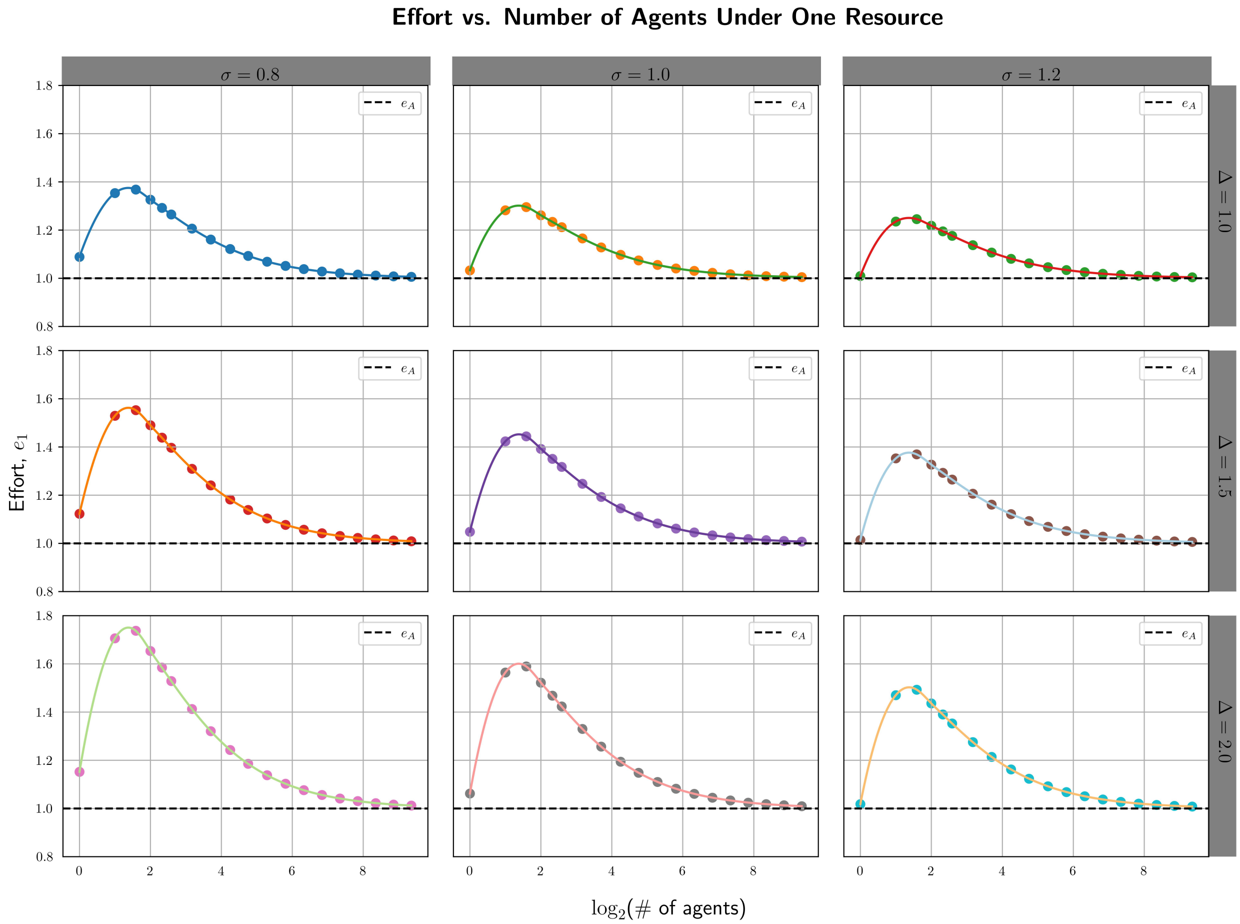

Figure 3 **The effect of competition on agents' effort: Agents' effort increases in Δ (traverse along rows), and decreases in σ (traverse along columns). Ceteris paribus, at low (resp., high) values of n, agents' effort is increasing (resp., decreasing) in n. Recall from (12) that the effort e_A maximizes an agent's period-1 payoff. Parameter values for this figure are: $\gamma = 0.2, \beta = 1, c = 1$.**

8.2. Endowment of Resources

For a given population size of the beneficiaries, say n, we analyze the effect of the endowment size of the superior resources m on the equilibrium outcome. Let y_{j1} denote the period-1 outcome of agent j, $j \in [n]$. Let $\widehat{\alpha}_{ij}(\boldsymbol{y})$ denote the probability that agent j receives superior resource i, $i \in [m]$. As before, let $\mathsf{rank}(j)$ denote the rank of agent j in the decreasing order of period-1 output (ties broken arbitrarily). We assume that the principal uses a "ranked-threshold" contract, that we describe below. Let $\tau \in \mathbb{R}$ denote the threshold that the principal chooses. Under the "ranked-threshold" contract, the principal adopts the following resource allocation strategy.

$$\widehat{\alpha}_{ij}(\boldsymbol{y}) = \begin{cases} 1, \text{ if } \mathsf{rank}(j) = i \text{ and } y_{j1} \geq \tau; \\ 0, \text{ o/w.} \end{cases}$$

Put simply, the principal gives the superior resource to the m agents with the highest period-1 output, among those whose output exceeds the threshold τ.

Suppose that the principal chooses a threshold τ, all other agents choose effort e_1, and agent j chooses e_{j1}. The probability that agent j receives resource i is:

$$\mathbb{P}\left[\text{agent } j \text{ receives resource } i \middle| e_{j1}, e_1\right] = \int_{y_{j1}=\tau}^{\infty} \psi\left(y_{j1}\middle|e_{j1}\right) \binom{n-1}{i-1} \left(1-\Psi\left(y_{j1}\middle|e_1\right)\right)^{i-1} \left(\Psi\left(y_{j1}\middle|e_1\right)\right)^{n-i} dy_{j1},$$

For e_1 to be a symmetric subgame equilibrium, we require the following:

$$e_1 \in \arg\max_{e_{j1}\geq 0} \left\{ u_1(e_{j1}) + \Delta \sum_{i=1}^{m} \mathbb{P}\left[\text{agent } j \text{ receives resource } i \middle| e_{j1}, e_1\right] \right\}. \tag{24}$$

The principal's problem simplifies to the following:

$$\max_{\tau\in\mathbb{R}} V = nv_1(e_1) + \Delta\left(2 - \frac{\gamma}{\beta}\right)\left(1 - s^n\left(\frac{1-\left(\frac{1-s}{s}\right)^m}{1-\left(\frac{1-s}{s}\right)}\right)\right)$$
$$\text{where } s = \mathbb{P}\left[y_{j1} \leq \tau \middle| e_1\right] = \Psi\left(\tau\middle|e_1\right) \text{ and } e_1 \text{ solves (24).}$$

Solving this problem analytically is challenging, and hence we optimize numerically. For a fixed population of agents (i.e., fixed value of n), Figure 4 below illustrates the impact of the endowment of superior resources (i.e., m) on agents' effort.

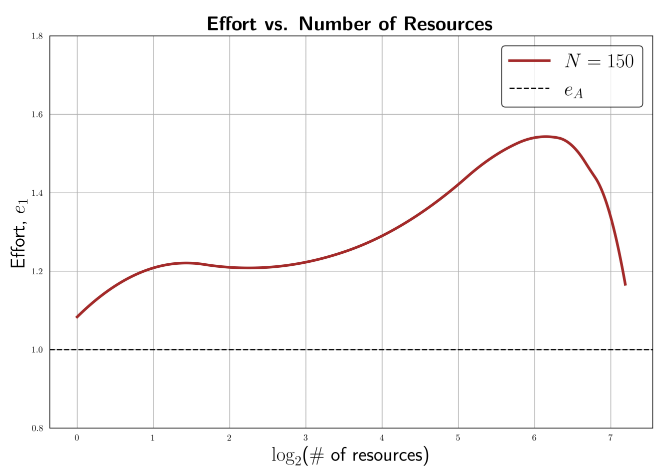

Figure 4 **The effect of the endowment of superior resources on agents' effort, for a fixed agent population: Agents' effort is increasing in m at low values of m, and is decreasing in m at high values of m. Recall from (12) that the effort e_A maximizes an agent's period-1 payoff. The parameter values for this figure are $\gamma = 0.2, \beta = 1, c = 1$.**

Insight: Optimal Amount of Scarcity

Observe from Figure 4 that at high levels of scarcity (low m), agents' effort is increasing in m. However, at low levels of scarcity (high m), agents' effort is decreasing in m. This signifies an important takeaway: *From the viewpoint of the NPO, for a fixed population size of the beneficiaries, the extent of scarcity of the superior resource to induce the highest effort from the beneficiaries is neither too low nor too high.*

8.3. Managing Multi-Beneficiary Pools: Dedicated vs. Pooled Resources

NPO's often design their initiatives for multiple groups of beneficiaries. For example, Tata Trusts' Karta Initiative operates in more than thirty schools and across six states in India (Horizons 2023). A key question that NPO managers who design such initiatives face while managing multiple beneficiary pools is whether they should earmark resources for each beneficiary groups, or should all beneficiaries be pooled together? To analyze this question, we consider two beneficiary groups of equal size, say n agents. We assume that the NPO is endowed with two superior resources ($m = 2$). We analyze the outcomes under the following two scenarios:

(a) Earmarking Dedicated Resources for Each Beneficiary Group: Two distinct groups, each consisting of $m = 1$ superior resource and a population of n agents.

(b) Pooling Both Beneficiary Groups: One (pooled) group, consisting of $m = 2$ superior resources and a population of $2n$ agents.

We adopt the procedure described in Sections 8.1 and 8.2 to compute the (symmetric) equilibrium effort of the agents and the principal's threshold. We illustrate the agents' effort in Figure 5 below.

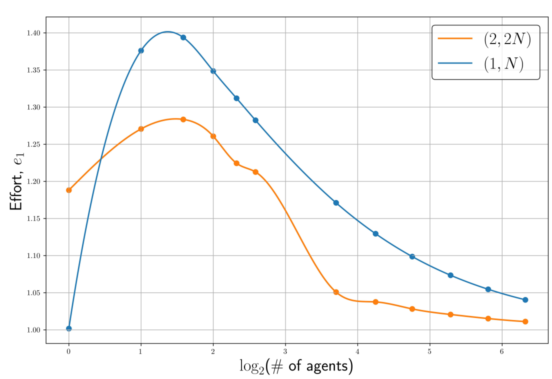

Figure 5 **Dedicated vs. Pooling Resources in Managing Multi-Beneficiary Pools: The blue curve corresponds to the agents' (symmetric) equilibrium effort when the NPO earmarks dedicated resources to each group. The orange curve corresponds to the agents' (symmetric) effort when the NPO manages the beneficiaries as one pool.**

Insight: To Earmark or Not to Earmark?

Recall from Sections 8.1 and 8.2 that the number of resources available and the extent of competition affect the agents' effort. Pooling leads to an increase in the number of resources available but also increases the competition for these resources, while earmarking dedicated resources for beneficiary subgroups leads to fewer available resources but also reduces competition for these resources. As shown in Figure 5, we find that at low levels of competition (low n), the availability effect dominates, and hence pooling induces greater effort. At high levels of competition (high n), the competition effect dominates and hence, earmarking dedicated resources induces greater effort. This observation informs the design of such initiatives: *When faced with scarce resources, the NPO should earmark dedicated resources and operate multiple beneficiary subgroups separately. When scarcity is less of a concern, the NPO should pool all beneficiary subgroups.*

9. Extension: Heterogenous Agent Types

Our base model assumes that agents are homogenous in their cost of effort. We now extend the model to allow for heterogenous agents who differ in their cost of effort. Specifically, let Θ denote the set of agent types, let $\theta \in \Theta$ denote an agent's (private) type, and let c_θ denote the agent's (private) cost parameter. For simplicity, we consider the case of two types: $\Theta = \{H, L\}$, where $0 < c_L < c_H$ and $\Phi = \{B, S\}$. The payoff of a type-θ-agent is as shown in (10), with c replaced by c_θ. The principal's payoff if the agent is of type θ is as shown in (11), with c replaced by c_θ.

We note that this setting involves both moral hazard and adverse selection. A well-known approach in the literature for this setting involves the use of screening mechanisms in which the principal offers an agent a menu of contracts and each agent self-selects her choice – thus, the principal deduces the type of each agent from that agent's preference within her menu (Maskin and Riley 1984). However, given our application context, namely an NPO supporting the education of underprivileged students, such a solution would involve offering contracts that differ with the types of the agents, and will therefore almost certainly be unacceptable in practice. Therefore, we focus on obtaining an optimal "uniform" contract, where the principal offers one contract to agents of all types. For instance, the NPOs we mentioned in Section 1 offer the same version of their support schemes to all intended beneficiaries.

9.1. Uniform Contract: Analysis

In the analysis below, we adopt the following notation. To denote a quantity for type θ, we append the subscript θ to our earlier notation. For example, the period-t effort for type θ is denoted by $e_{\theta t}$. For the equilibrium quantities obtained in Section 5 (where the analysis was under complete information), we also add the superscript ci. For example, the period-1 effort under complete information for a type-θ-agent is denoted by $e^{\mathsf{ci}}_{\theta 1}$.

As before, the agent's period-2 problem is straightforward. From the analysis in Section 3.4, the period-2 effort for an agent of type θ under resource ϕ is $\widehat{e}_{\theta 2}(\phi) = e^{\star}_{\theta 2}(\phi) = \frac{k_\phi}{2\beta c_\theta}$. Using this, we write the principal's problem of obtaining an optimal uniform contract under moral hazard and adverse selection as follows:

$$\left.\begin{aligned} \max_{\widehat{\alpha}(\cdot),\{\widehat{e}_{\theta 1}\}_{\theta\in\Theta}} V &= \mathbb{E}_\theta\left[v(\widehat{e}_{\theta 1};\theta) + \left(2-\frac{\gamma}{\beta}\right)\Delta_\theta \int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_{\theta 1}\right)\right] \\ \text{s.t. } \widehat{e}_{\theta 1} &\in \arg\max_{e_1\ge 0}\; u_{\theta 1}(e_1) + \Delta_\theta \int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|e_1\right) \quad \forall\theta\in\Theta. \end{aligned}\right\} \quad (\textsc{Problem AS} - \textsc{Uniform})$$

Definition 2. A *uniform threshold contract*, with threshold $\tau\in\mathbb{R}$ is defined as follows:

1. The recommended period-1 effort to a type-θ-agent, $\widehat{e}_{\theta 1}$, is the solution to (15), with e_A replaced by $e_{A\theta}$.
2. The principal's resource-allocation strategy, $\widehat{\alpha}(\cdot)$, is as shown in (14), for some $\tau\in\mathbb{R}$.
3. The recommended period-2 effort strategy to a type-θ-agent, $\widehat{e}_{\theta 2}$, is as shown in (8), with c replaced by c_θ.

Theorem 8. *There exists an optimal uniform contract that is a threshold contract. That is, there exists an optimal strategy for the principal where the resource-allocation strategy $\widehat{\alpha}(\cdot)$ has the following structure:*

$$\widehat{\alpha}(y) = \begin{cases} 0, & \textit{if } y<\tau; \\ 1, & \textit{if } y\ge\tau. \end{cases}$$

for some $\tau\in\mathbb{R}$. Further, $\tau^{\mathsf{ci}}_H \le \tau \le \tau^{\mathsf{ci}}_L$, where τ^{ci}_H (resp., τ^{ci}_L) is the equilibrium value of the threshold τ for a type-H (resp., type-L) agent in the complete-information setting in Section 5.

Below, we write the principal's problem for identifying the optimal threshold.

$$\begin{aligned} \max_{\tau^{\mathsf{ci}}_H\le\tau\le\tau^{\mathsf{ci}}_L} V &= \mathbb{E}_\theta\left[v(e_{\theta 1};\theta) + \left(2-\frac{\gamma}{\beta}\right)\Delta_\theta\left(1-\Psi\left(\tau\middle|e_{\theta 1}\right)\right)\right] \\ \text{where } e_{\theta 1} &= e_{A\theta}\left(1+\Delta_\theta\psi\left(\tau\middle|e_{\theta 1}\right)\right) \text{ for each } \theta\in\Theta. \end{aligned}$$

The constraint above – the best response of a type-θ agent to a threshold τ – follows from (15). In general, V is not concave in τ; hence, the first-order condition may not be sufficient. Nevertheless, the principal's problem for an optimal threshold simplifies to a one-dimensional search in the interval $[\tau^{\mathsf{ci}}_H, \tau^{\mathsf{ci}}_L]$.

10. Concluding Remarks

Arguably, the most important determinant of an NPO's effectiveness is its resource-allocation strategy. Our work in this paper focuses on NPOs in the education sector that adopt a two-stage structure in the allocation of resources to its beneficiaries. We demonstrate the strategic role of an NPO's resource-allocation strategy in incentivizing its target population of beneficiaries to exert effort. In particular, we show why and how an NPO restricting access of its resources to the beneficiaries can result in superior lifetime outcomes for both the beneficiaries and the NPO. Further, our results shed light on several operational decisions that managers who design and administer such initiatives face, e.g., the size of the beneficiary population, the size of the endowment of resources, and the value of pooling vs. earmarking dedicated resources in managing multi-beneficiary pools.

Beyond non-profit organizations, our work is of relevance to many contexts common in OM. In particular, in many operational environments, non-monetary incentives are employed by organizations (e.g., firms, buyers) to induce agents (e.g., workers, suppliers) to exert effort. In such settings, the effort exerted by agents is a strategic complement to non-monetary incentives (investments) provided by the organization. For example, firms routinely invest in worker training (e.g., upskilling), and buyers invest in improving suppliers' manufacturing technology (e.g., machinery, personnel, etc.). Despite the seemingly-beneficial role of such investments (resources) and despite their availability, we demonstrate the importance of restricting access to such resources, and how the response from the beneficiaries changes with the extent of competition for resources and the size of the population of beneficiaries.

Our work contributes to the growing literature on socially-responsible OM, operations in developing economies, and non-profit OM. Apart from an NPO's resource-allocation strategy, there are several other key operational factors that determine the NPO's effectiveness, such as ensuring adequate funds and resources, managing donors, outreach and communication, workforce planning, recruiting and reducing staff turnover, technology modernization and innovation, improving governance by engaging board members, etc. Indeed, there has been a growing interest within the academic OM community to address practical operational challenges in these and other related topics; see, for example, Natarajan and Swaminathan (2014), Devalkar et al. (2017), Ata et al. (2019), Arora et al. (2022), Zhang et al. (2022), Virudachalam et al. (2023). We believe that there are tremendous opportunities to further develop this stream of research.

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Online Appendix for "Non-Profit Support in Education: Resource Allocation and Students' Lifetime Outcomes"

Appendix A: Proofs of Results in the Main Paper

For convenience of notation, we let $\psi_0(\cdot)$ and $\Psi_0(\cdot)$ denote the p.d.f. and c.d.f., respectively, of the standard normal distribution, i.e., $\mathcal{N}(0,1)$.

A.1. Proofs of Results in Section 4

Proof of Theorem 1: Recall the definition of e_A from (12). Under Free, we have $\widehat{\alpha}(y)=1$ for all $y\in\mathbb{R}$. Substituting for $\widehat{\alpha}(\cdot)$ in (10), the agent's expected payoff simplifies to

$$U(e_1)=\underbrace{\left(e_1-\beta\frac{c}{2}e_1^2\right)}_{u_1(e_1)}+u_2^\star(S).$$

That is, the second-period payoff is $u_2^\star(S)$ and is independent of e_1. Consequently, the agent's best response, i.e., the effort that maximizes the period-1 payoff, is $e_1^\star=e_A$.

Q.E.D.

A.2. Proofs of Results in Section 5

Proof of Theorem 2: Consider any $\widehat{\alpha}(\cdot)$. We write the agent's problem (ignoring the constant terms) in Problem MH below.

$$\max_{e_1\geq 0}\left(e_1-\beta\frac{c}{2}e_1^2\right)+\Delta\int_{y_1\in\mathbb{R}}\widehat{\alpha}(y_1)\psi\left(y_1\middle|e_1\right)dy_1.$$

Since the agent's payoff is smooth, the first-order condition must hold at optimality. Thus, the agent's best response must satisfy:

$$1-\beta c e_1+\Delta\int_{y_1\in\mathbb{R}}\widehat{\alpha}(y_1)\psi\left(y_1\middle|e_1\right)\left(\frac{y_1-e_1}{\sigma^2}\right)dy_1=0.$$

Rearranging the above, we have the following fixed-point equation:

$$\begin{aligned}
e_1 &= e_A\left(1+\frac{\Delta}{\sigma^2}\underbrace{\int_{y_1\in\mathbb{R}}\widehat{\alpha}(y_1)\psi\left(y_1\middle|e_1\right)(y_1-e_1)}_{\mathbb{E}[\widehat{\alpha}(y_1)(y_1-e_1)|e_1]}\right)\\
&= e_A\left(1+\frac{\Delta}{\sigma^2}\mathrm{Cov}\left(\widehat{\alpha}(y_1)\middle|e_1,y_1\middle|e_1\right)\right).
\end{aligned}$$

From Lemma B.1, it follows that smallest (resp., largest) value of the right-hand-side is $\underline{e}$ (resp., $\overline{e}$). Consequently, for any $e_1\notin\mathcal{E}=[\underline{e},\overline{e}]$, e_1 cannot be a solution to the equation above, and hence cannot be induced.

Next, consider any $e_1\in\mathcal{E}$. We will identify a contract that induces e_1.

Case (i): Suppose $e_1\geq e_A$. Then, the "threshold" contract, as identified in Lemma 2, induces e_1 (see proof of Lemma 2 below).

Case (ii): Suppose $e_1 < e_A$. Then, for a fixed $\hat{\tau}$ (that we will define below), consider an "inverted threshold" contract, as follows:

$$\widehat{\alpha}(y) = \begin{cases} 1, \text{ if } y \leq \widehat{\tau}; \\ 0, \text{ o/w.} \end{cases}$$

Consider the agent's best response to the above inverted-threshold contract. The agent's best response must satisfy the following first-order condition:

$$1 - \beta c e_1 - \Delta\psi\left(\widehat{\tau}\middle| e_1\right) = 0.$$

Rearranging the above, the agent's best response is the solution to the following fixed-point equation:

$$e_1 = e_A\left(1 - \Delta\psi\left(\widehat{\tau}\middle| e_1\right)\right).$$

The best response above is unique due to Assumption 1. Stated differently, consider any $e_1 \in [\underline{e}, e_A)$. Then, an inverted-threshold contract, with $\widehat{\tau} = e_1 + \psi^{-1}_{(-)}\left(\frac{1}{\Delta}\left(\frac{e_A - e_1}{e_A}\right)\right)$ or $\widehat{\tau} = e_1 + \psi^{-1}_{(+)}\left(\frac{1}{\Delta}\left(\frac{e_A - e_1}{e_A}\right)\right)$, induces e_1.

Q.E.D.

Proof of Lemma 1: Consider $\alpha(\cdot)$ as shown in (14). The agent's best-response satisfies the first-order condition and can be written as follows:

$$1 - \beta c e_1 - \Delta\psi\left(\tau\middle| e_1\right) = 0 \implies e_1 = e_A\left(1 + \Delta\psi\left(\tau\middle| e_1\right)\right). \tag{A.25}$$

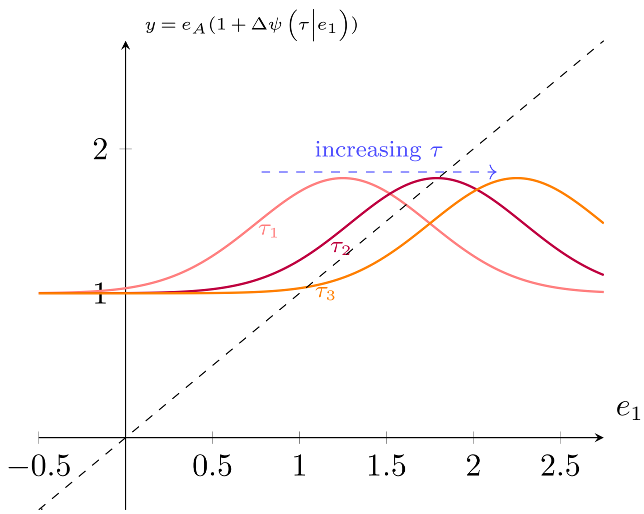

Figure 6 Demonstrating the fixed point equation in (A.25). Values of Parameters: $c = 1, \beta = 1, \Delta = 1, \sigma = 0.5$. **The value of** $e_A = 1$ **and** $\overline{e} \approx 1.79$. **The red curve corresponds to** $\tau = 1.25$, **purple curve corresponds to** $\tau = \overline{e}$, **and the orange curve corresponds to** $\tau = 2.25$.

Note that (A.25) is a fixed-point equation. Figure 6 demonstrates this fixed-point equation for three different values of τ. First, consider any $\tau \in \mathbb{R}$ (e.g., consider one of the "curves" in Figure 6). We show that the solution to the fixed-point equation in (A.25) is unique under Assumption 1 (i.e., the point of intersection of the curve and the dashed line in Figure 6 is unique). Using Lemma B.2, we have that the maximum slope of the right-hand-side of (A.25) is $e_A\left(\frac{\Delta}{\sqrt{2\pi\sigma^2}}\right)$. From Assumption 1, this maximum slope is less than 1. Consequently, $e_A\left(1+\Delta\psi\left(\tau\middle|e_1\right)\right)-e_1$ is decreasing in e_1. Furthermore, we have:

(a) At $e_1 = e_A$, the left-hand-side of (A.25) is e_A, while the right-hand-side of (A.25) is strictly larger than e_A; hence, $\left(e_A\left(1+\Delta\psi\left(\tau\middle|e_1\right)\right)-e_1\right)\Big|_{e_1=e_A} > 0$.

(b) In the limit (i.e., as $e_1 \to \infty$), the left-hand-side of (A.25) approaches ∞, while the right-hand-side of (A.25) approaches e_A; hence, $\lim_{e_1\to\infty}\left(e_A\left(1+\Delta\psi\left(\tau\middle|e_1\right)\right)-e_1\right) < 0$

It now follows immediately that the solution to (A.25) is unique, and hence the first-order condition identifies the agent's unique best response.

The right-hand-side of (A.25) is bounded from above by $\bar{e}$ and bounded from below by e_A. From the above, we have the following:

$$\lim_{\tau\to\pm\infty} e_1 = e_A$$
$$e_1\Big|_{\tau=\bar{e}} = \bar{e}$$

Hence, the maximum (resp., minimum) value of e_1 that can be induced is $\bar{e}$ (resp., e_A).

Consider any $z \in \mathbb{R}$, and consider the following value of τ:

$$\tau \equiv \tau(z) = e_A\left(1+\frac{\Delta}{\sigma}\psi_0(z)\right)+\sigma z. \tag{A.26}$$

Observe that τ is strictly increasing in z due to Assumption 1, with the following:

$$\lim_{z\to-\infty}\tau = -\infty,\quad \lim_{z\to\infty}\tau = \infty,\ \tau\Big|_{z=0} = \bar{e}.$$

The agent's best response to τ in (A.26), i.e., the solution to (A.25) for τ in (A.26), is the following:

$$e_1 = e_A\left(1+\frac{\Delta}{\sigma}\psi_0(z)\right). \tag{A.27}$$

In particular, e_1 is increasing in z for $z<0$ and decreasing in z for $z>0$. Therefore, e_1 is increasing in τ if $\tau<\bar{e}$, and decreasing in τ if $\tau>\bar{e}$.

Q.E.D.

Proof of Lemma 2: Consider τ as shown in (A.26). The best response is as shown in (A.27). It follows from the proof of Lemma 1 that for any effort e_1 in the set $(e_A,\bar{e}]$, there exists a finite z, and hence a τ that induces e_1. Further, Free induces e_A.

Next, consider any $z\in\mathbb{R}$. Observe that the induced effort at $\tau=\tau(z)$ and $\tau=\tau(-z)$ (where τ is as shown in (A.26)) is identical. Stated differently, fix e_1, and consider the following values of z:

$$z_{(-)} = \psi^{-1}_{0(-)}\left(\frac{\sigma}{\Delta}\left(\frac{e_1-e_A}{e_A}\right)\right) \text{ and } z_{(+)} = \psi^{-1}_{0(+)}\left(\frac{\sigma}{\Delta}\left(\frac{e_1-e_A}{e_A}\right)\right)$$

Substituting the above values in the expression for τ in (A.26), we obtain the two thresholds as follows:

$$\tau_{(-)} = e_1+\sigma z_{(-)} \text{ and } \tau_{(+)} = e_1+\sigma z_{(+)} \tag{A.28}$$

Q.E.D.

Proof of Lemma 3: Recall the principal's payoff under the threshold contract in (18). Consider any $e_1 \in \mathcal{E}$, $e_1 < \overline{e}$. The two thresholds that induce e_1 are as shown in (A.28). Since $\tau_{(-)}$ and $\tau_{(+)}$ induce the same effort, the principal's period 1 payoff is identical. In period 2, observe that the principal's payoff is increasing in the probability that the agent receives the superior resource. Since this probability is (strictly) higher under $\tau_{(-)}$, i.e.,

$$\underbrace{1-\Psi\left(\tau_{(-)}\middle|e_1\right)}_{\mathbb{P}[\phi=S|\tau_{(-)}]} > \underbrace{1-\Psi\left(\tau_{(+)}|e_1\right)}_{\mathbb{P}[\phi=S|\tau_{(+)}]},$$

we have that $V(\tau_{(-)}) > V(\tau_{(+)})$. Finally, if $e_1 = \overline{e}$, we have that $\tau_{(-)} = \tau_{(+)} = \overline{e}$; here $V(\tau_{(-)}) = V(\tau_{(+)})$.

Q.E.D.

Proof of Theorem 3: Consider any $\widehat{e}_1 \in \mathcal{E} = [\underline{e}, \overline{e}]$ that the principal chooses to induce. Consider the problem that the principal faces:

$$\max_{\widehat{\alpha}(\cdot)} V = v_1(\widehat{e}_1) + \Delta\left(2-\frac{\gamma}{\beta}\right)\int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_1\right)dy \tag{A.29}$$

$$\text{s.t. } \widehat{e}_1 \in \arg\max_{e_1\geq 0}\ u_1(e_1) + \Delta\int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|e_1\right)dy. \tag{A.30}$$

We relax the above problem by replacing the constraint with the corresponding first-order condition. That is, we replace (A.30) by the corresponding first-order condition:

$$\underbrace{1-\beta c\widehat{e}_1}_{u_1'(\widehat{e}_1)} + \Delta\int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_1\right)\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)dy = 0. \tag{A.31}$$

Indeed, the optimal solution to the problem in (A.29)-(A.30) must satisfy (A.31). Observe that in the first-order condition (to the agent's problem), we have interchanged the integral and the differential operators (since the derivative of the Gaussian p.d.f. is bounded from above by 1). By replacing (A.30) with (A.31), we have the following optimization problem:

$$\begin{aligned}
\max_{\widehat{\alpha}(\cdot)}\ V &= v_1(\widehat{e}_1) + \Delta\left(2-\frac{\gamma}{\beta}\right)\int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_1\right)dy \\
\text{s.t.} \quad &(1-\beta c\widehat{e}_1) + \Delta\int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_1\right)\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)dy = 0.
\end{aligned}$$

The Lagrangian can be written as follows:

$$\begin{aligned}
\mathcal{L} &= \left(v_1(\widehat{e}_1) + \Delta\left(2-\frac{\gamma}{\beta}\right)\int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_1\right)dy\right) + \lambda\left(1-\beta c\widehat{e}_1 + \Delta\int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_1\right)\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)dy\right) \\
&= \int_{y\in\mathbb{R}} \psi\left(y\middle|\widehat{e}_1\right)\left(\underbrace{\underbrace{v_1(\widehat{e}_1)+\lambda(1-\beta c\widehat{e}_1)}_{\text{independent of } \widehat{\alpha}(y)} + \widehat{\alpha}(y)\left(\underbrace{2-\frac{\gamma}{\beta}+\lambda\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)}_{\spadesuit}\right)\Delta}_{\text{maximize ``pointwise''}}\right)dy.
\end{aligned}$$

We now maximize the right-hand-side "pointwise". That is, for each $y \in \mathbb{R}$, we maximize the term inside the brackets in the integrand. The first term inside the brackets is independent of $\widehat{\alpha}(y)$. The second term inside is linear in $\widehat{\alpha}(y)$, with the coefficient $\spadesuit\Delta$. Thus, the optimal decision is as follows:

$$\widehat{\alpha}(y) = \begin{cases} 1, \text{ if } \spadesuit\left(= 2 - \frac{\gamma}{\beta} + \lambda\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)\right) \geq 0; \\ 0, \text{ if } \spadesuit\left(= 2 - \frac{\gamma}{\beta} + \lambda\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)\right) < 0. \end{cases} \tag{A.32}$$

It remains to be seen whether λ is positive or negative.

(i) Suppose $\lambda < 0$. Substituting (A.32) in (A.31), we get:

$$\int_{-\infty}^{\widehat{e}_1 - \frac{\sigma^2}{\lambda}\left(2-\frac{\gamma}{\beta}\right)} \psi\left(y\middle|\widehat{e}_1\right)\left(\frac{y-\widehat{e}_1}{\sigma^2}\right) dy = -\frac{u_1'(\widehat{e}_1)}{\Delta}.$$

$$\Longrightarrow \underbrace{-\int_{-\infty}^{-\left|\frac{\sigma}{\lambda}\right|\left(2-\frac{\gamma}{\beta}\right)} \psi_0\left(\widehat{y}\right)\widehat{y}d\widehat{y}}_{\text{positive}} = \frac{u_1'(\widehat{e}_1)}{\Delta}.$$

Since the left-hand-side is positive, it must be that $\widehat{e}_1 < e_A$.

(ii) Suppose $\lambda > 0$. Applying a similar technique, i.e., substituting (A.32) in (A.31), we get:

$$\underbrace{-\int_{\frac{\sigma}{\lambda}\left(2-\frac{\gamma}{\beta}\right)}^{\infty} \psi_0(\widehat{y})\widehat{y}d\widehat{y}}_{\text{negative}} = \frac{u_1'(\widehat{e}_1)}{\Delta}.$$

Since the left-hand-side is negative, it must be that $\widehat{e}_1 > e_A$.

Since the relaxation yields a unique solution that is feasible to the original problem (in (A.29)-(A.30)), it is optimal to (A.29)-(A.30). Hence, to induce any effort $\widehat{e}_1 \in \mathcal{E}$, it suffices to restrict attention to threshold contracts. Further, the following holds: For any $t \in \mathbb{R}$,

$$\int_t^{\infty} z\psi_0(z)dz = \psi_0(t).$$

Using (ii), for any $\widehat{e}_1 \in \mathcal{E}$, $\widehat{e}_1 \geq e_A$, we can write the corresponding condition as follows:

$$\psi_0\left(\frac{\sigma}{\lambda}\left(2-\frac{\gamma}{\beta}\right)\right) = -\frac{\sigma}{\Delta}u_1'(\widehat{e}_1). \tag{A.33}$$

Since $\lambda > 0$, observe that the condition in (A.32) can be written as follows:

$$\widehat{\alpha}(y) = \begin{cases} 1, \text{ if } y > \tau; \\ 0, \text{ o/w.} \end{cases}$$

$$\text{where } \tau = \widehat{e}_1 - \frac{\sigma^2}{\lambda}\left(2-\frac{\gamma}{\beta}\right).$$

Substituting (A.33) in the above, the threshold τ can be written as:

$$\tau = \widehat{e}_1 + \sigma \underbrace{\psi_{0(-)}^{-1}\left(\frac{\sigma}{\Delta}\left(\frac{\widehat{e}_1 - e_A}{e_A}\right)\right)}_{z_{(-)}}. \tag{A.34}$$

That is, to induce an effort $\widehat{e}_1 \geq e_A$, $\widehat{e}_1 \in \mathcal{E}$, it is optimal for the principal to use the "threshold" lottery $\widehat{\alpha}(\cdot)$ with the threshold shown in (A.34). Q.E.D.

Proof of Lemma 4: To show part (a), observe that from (A.28), we have:

$$\tau(\widehat{e}_1) - \widehat{e}_1 = \sigma z_{(-)} = \sigma \underbrace{\psi_{0(-)}^{-1}\left(\frac{\sigma}{\Delta}\left(\frac{\widehat{e}_1 - e_A}{e_A}\right)\right)}_{z_{(-)}}.$$

The right-hand-side above is increasing in $\widehat{e}_1 \in \mathcal{E} = [\underline{e}, \overline{e}]$.

Next, we show part (b). Recall the expression for $V(\widehat{e}_1)$ in (18). The period-1 payoff is concave in $\widehat{e}_1$. We focus on the period-2 payoff. We ignore the constant terms and focus on the last term $1 - \Psi\left(\tau(\widehat{e}_1)\middle|\widehat{e}_1\right)$, which corresponds to the probability that the agent receives the superior good under $\widehat{e}_1$. This quantity is denoted by $\mathbb{P}\left[\phi = S\middle|\widehat{e}_1\right]$. Therefore, it suffices to show that $\mathbb{P}\left[\phi = S\middle|\widehat{e}_1\right]$ is concave in $\widehat{e}_1$.

$$\begin{aligned}
\frac{d}{d\widehat{e}_1}\left(\mathbb{P}[\phi = S\middle|\widehat{e}_1]\right) &= \frac{d}{d\widehat{e}_1}\left(1 - \Psi\left(\tau(\widehat{e}_1)\middle|\widehat{e}_1\right)\right) \\
&= \frac{d}{d\widehat{e}_1}\left(1 - \Psi_0\left(\psi_{0(-)}^{-1}\left(\frac{\sigma}{\Delta}\left(\frac{\widehat{e}_1 - e_A}{e_A}\right)\right)\right)\right) \\
&= -\underbrace{\frac{\sigma}{\Delta}\left(\frac{\widehat{e}_1 - e_A}{e_A}\right)(\psi_{0(-)}^{-1})'\left(\frac{\sigma}{\Delta}\left(\frac{\widehat{e}_1 - e_A}{e_A}\right)\right)}_{\widehat{g}\left(\frac{\sigma}{\Delta}\left(\frac{\widehat{e}_1 - e_A}{e_A}\right)\right)}\frac{\sigma}{\Delta e_A}.
\end{aligned} \tag{A.35}$$

Observe that the right-hand-side in (A.35) is negative; thus $\mathbb{P}\left[\phi = S\middle|\widehat{e}_1\right]$ is decreasing in $\widehat{e}_1$. Next, define $\widehat{g}(\cdot)$ as follows:

$$\widehat{g}(z) = z(\psi_{0(-)}^{-1})'(z). \tag{A.36}$$

The right-hand-side in (A.35) can be written as $-\widehat{g}\left(\frac{\sigma}{\Delta}\left(\frac{\widehat{e}_1 - e_A}{e_A}\right)\right)\frac{\sigma}{\Delta e_A}$. To show that $\mathbb{P}\left[\phi = S\middle|\widehat{e}_1\right]$ is concave in $\widehat{e}_1$, it suffices to show that $\widehat{g}(\cdot)$ is an increasing function. We show this in Lemma B.3. Our result then follows.

Q.E.D.

Proof of Theorem 4: Using the first-order condition from (19), we have:

$$\begin{aligned}
V'(\widehat{e}_1) = 0 &\implies 1 - \frac{\widehat{e}_1}{e_P} = \frac{\left(2 - \frac{\gamma}{\beta}\right)\sigma}{e_A\sqrt{2\log\left(\frac{\overline{e} - e_A}{\widehat{e}_1 - e_A}\right)}}. \\
&\implies \log\left(\frac{\overline{e} - e_A}{\widehat{e}_1 - e_A}\right) = \frac{1}{2}\left(\frac{\left(2 - \frac{\gamma}{\beta}\right)\frac{\sigma}{e_A}}{1 - \frac{\widehat{e}_1}{e_P}}\right)^2. \\
&\implies \frac{\overline{e} - e_A}{\widehat{e}_1 - e_A} = \exp\left(\frac{1}{2}\left(\frac{\left(2 - \frac{\gamma}{\beta}\right)\frac{\sigma}{e_A}}{1 - \frac{\widehat{e}_1}{e_P}}\right)^2\right). \\
&\implies \widehat{e}_1 = e_A + (\overline{e} - e_A)\exp\left(-\frac{1}{2}\left(\frac{\left(2 - \frac{\gamma}{\beta}\right)\frac{\sigma}{e_A}}{1 - \frac{\widehat{e}_1}{e_P}}\right)^2\right).
\end{aligned}$$

Q.E.D.

A.3. Proofs of Results in Section 6

Proof of Theorem 5: Using the definition of $\overline{e}$, we rewrite the induced effort in (20) below:

$$\begin{aligned}\widehat{e}_1 &= e_A + (\overline{e} - e_A)\exp\left(-\frac{1}{2}\left(\frac{\left(2-\frac{\gamma}{\beta}\right)\frac{\sigma}{e_A}}{1-\frac{\widehat{e}_1}{e_P}}\right)^2\right)\\ &= e_A\left(1+\frac{\Delta}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{\left(2-\frac{\gamma}{\beta}\right)\frac{\sigma}{e_A}}{1-\frac{\widehat{e}_1}{e_P}}\right)^2\right)\right). \end{aligned} \tag{A.37}$$

We show each part below. Recall from Theorem 4 that the fixed-point equation (A.37) has a unique solution.

(a) Consider the right-hand-side in (A.37). Recall that this expression is decreasing in $\widehat{e}_1$ and also decreasing in σ. Hence, the solution to $\widehat{e}_1$ in (A.37) is decreasing in σ.

(b) Similar to (a), the right-hand-side is decreasing in c since e_A and Δ are decreasing in c. Hence, the solution to $\widehat{e}_1$ in (A.37) is decreasing in c.

(c) Similar to (a), the right-hand-side is increasing in k_S and decreasing in k_B, since Δ is increasing in k_S and decreasing in k_B. Hence, the solution to $\widehat{e}_1$ in (A.37) is increasing in $\frac{k_S}{k_B}$.

(d) To show the comparative static of $\widehat{e}_1$ with respect to γ, we first observe that the term $\frac{(2-\frac{\gamma}{\beta})\frac{\sigma}{e_A}}{1-\frac{\widehat{e}_1}{e_P}}$ is increasing in γ. To see this, observe that:

$$\frac{d}{d\gamma}\left(\frac{(2-\frac{\gamma}{\beta})\frac{\sigma}{e_A}}{1-\frac{\widehat{e}_1}{e_P}}\right) = \frac{\sigma c}{\left(1-\frac{\widehat{e}_1}{e_P}\right)^2}\left(2\frac{\widehat{e}_1}{e_A}-1\right) > 0.$$

The right-hand-side of (A.37) is decreasing in this term; hence $\widehat{e}_1$ is decreasing in γ. To show the comparative static of $\widehat{e}_1$ with respect to β, observe that the right-hand-side is decreasing in β; hence, $\widehat{e}_1$ is decreasing in β.

Q.E.D.

Proof of Theorem 6: Recall from (22) that the probability that the agent receives the good is:

$$\mathbb{P}[\phi = S]\Big|_{\widehat{e}_1} = 1-\Psi_0\left(\psi^{-1}_{0(-)}\left(\frac{\sigma}{\Delta}\frac{\widehat{e}_1 - e_A}{e_A}\right)\right).$$

Let $\xi^\star$ denote the following:

$$\xi^\star \triangleq \frac{\sigma}{\Delta}\frac{\widehat{e}_1 - e_A}{e_A}.$$

The probability $\mathbb{P}[\phi = S] = 1-\Psi_0\left(\psi^{-1}_{0(-)}(\xi^\star)\right)$ is decreasing in $\xi^\star$. It suffices to analyze how $\xi^\star$ changes with the model parameters. We can write (A.37) as follows:

$$\xi^\star = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\left(2-\frac{\gamma}{\beta}\right)\sigma\beta c}{1-\frac{\gamma}{\beta}\left(1+\xi^\star\frac{k_S^2-k_B^2}{2\beta c\sigma}\right)}\right)^2\right).$$

Note that this is a fixed-point equation in $\xi^\star$. Let $c\sigma = \widehat{c}$, $\frac{\gamma}{\beta} = \widehat{\gamma}$, $\widehat{k} = \frac{k_S^2-k_B^2}{2}$. Then,

$$\xi^\star = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{(2-\widehat{\gamma})\beta\widehat{c}}{\underbrace{1-\widehat{\gamma}(1+\frac{\xi^\star\widehat{k}}{\beta\widehat{c}})}_{=\clubsuit}}\right)^2\right). \tag{A.38}$$

The right-hand-side of (A.38) is smooth and decreasing in $\xi^\star$. Consider any model primitive. The effect of the model primitive on $\xi^\star$ can be written as:

$$\frac{d\xi^\star}{d(\text{model primitive})} = \frac{\frac{\partial(\text{r.h.s. of (A.38)})}{\partial(\text{model primitive})}}{1-\frac{\partial(\text{r.h.s. of (A.38)})}{\partial\xi^\star}}. \tag{A.39}$$

Since the right-hand-side of (A.38) is decreasing in $\xi^\star$, the denominator of the right-hand-side of (A.39) is positive; hence, it suffices to analyze the numerator of the right-hand-side in (A.39). Further, the right-hand-side of (A.38) is decreasing in the quantity $\frac{(2-\widehat{\gamma})\beta\widehat{c}}{1-\widehat{\gamma}(1+\frac{\xi^\star\widehat{k}}{\beta\widehat{c}})}$, denoted by ♣. Hence, to analyze the numerator of the right-hand-side in (A.39), we analyze ♣.

To show part (a), we identify conditions such that ♣ is increasing in $\widehat{c}$.

$$\frac{d♣}{d\widehat{c}} = \frac{(2-\widehat{\gamma})\beta^2\widehat{c}}{\left(1-\widehat{\gamma}\left(1+\frac{\xi^\star\widehat{k}}{\beta\widehat{c}}\right)\right)^2}\left(1-\widehat{\gamma}\left(1+2\frac{\xi^\star\widehat{k}}{\beta\widehat{c}}\right)\right).$$

Recall that $\left(1-\widehat{\gamma}\left(1+\frac{\xi^\star\widehat{k}}{\beta\widehat{c}}\right)\right)>0$. However, we need a stronger condition for $\left(1-\widehat{\gamma}\left(1+2\frac{\xi^\star\widehat{k}}{\beta\widehat{c}}\right)\right)>0$. We consider the following sufficient condition:

$$\widehat{c} > \frac{2\widehat{\gamma}}{1-\widehat{\gamma}}\frac{\widehat{k}}{\beta}\frac{1}{\sqrt{2\pi}}.$$

The condition in the statement of Theorem 6 is equivalent to the condition above.

Part (b) is straightforward because ♣ is increasing in $\widehat{k}$; hence, the right-hand-side of (A.38) is decreasing in $\widehat{k}$. Consequently, $\xi^\star$ is decreasing in $\widehat{k}$, and hence $\mathbb{P}[\phi=S]$ is increasing in $\widehat{k}$.

In part (c), the comparative static with respect to γ is straightforward: Observe that

$$\frac{d♣}{d\widehat{\gamma}} = \frac{\beta\widehat{c}}{\left(1-\widehat{\gamma}\left(1+\frac{\xi^\star\widehat{k}}{\beta\widehat{c}}\right)\right)^2}\left(1+\frac{2\xi^\star\widehat{k}}{\widehat{c}}\right).$$

Since the right-hand-side is positive, the solution to $\xi^\star$ is decreasing in $\widehat{\gamma}$, and hence $\mathbb{P}[\phi=S]$ is increasing in $\widehat{\gamma}$. Consequently, $\mathbb{P}[\phi=S]$ is increasing in γ. Q.E.D.

A.4. Proofs of Results in Section 7

Proof of Theorem 7: Consider a set of resources Φ, where k_ϕ, denotes the efficacy of resource $\phi\in\Phi$. Consider the principal's problem as shown in Problem MH − Multiple. Consider an arbitrary $\widehat{e}_1\geq 0$. We identify an optimal contract that induces $\widehat{e}_1$. We adopt an identical "first-order" approach as in the proof of Theorem 3. That is, we replace the constraint in Problem MH − Multiple – the agent's optimization problem – with the corresponding first-order condition.

For any $\phi\in\Phi$, define Δ_ϕ as follows:

$$\Delta_\phi = \frac{k_\phi^2-k_{\underline{\phi}}^2}{2\beta c}.$$

The ratio Δ_ϕ (analogous to Δ) denotes the efficacy premium of resource ϕ relative to $\underline{\phi}$. After substituting the above in Problem MH − Multiple and ignoring the constant terms, the principal's problem is as follows:

$$\max_{\{\widehat{\alpha}(\phi|\cdot)\}_{\phi\in\Phi\setminus\{\underline{\phi}\}}} V = v_1(\widehat{e}_1)+\left(2-\frac{\gamma}{\beta}\right)\sum_{\phi\in\Phi\setminus\{\underline{\phi}\}}\Delta_\phi\int_{y\in\mathbb{R}}\widehat{\alpha}\left(\phi\middle|y\right)\psi\left(y\middle|\widehat{e}_1\right)dy$$

$$\text{s.t.}\quad u_1'(\widehat{e}_1)+\sum_{\phi\in\Phi\setminus\{\underline{\phi}\}}\Delta_\phi\int_{y\in\mathbb{R}}\widehat{\alpha}\left(\phi\middle|y\right)\psi\left(y\middle|\widehat{e}_1\right)\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)dy=0.$$

We write the Lagrangian below.

$$\begin{aligned}\mathcal{L} &= v_1(\widehat{e}_1) + \left(2-\frac{\gamma}{\beta}\right)\sum_{\phi\in\Phi\setminus\{\underline{\phi}\}}\Delta_\phi\int_{y\in\mathbb{R}}\widehat{\alpha}\left(\phi\middle|y\right)\psi\left(y\middle|\widehat{e}_1\right)dy\; + \\ &\quad \lambda\left(u_1'(\widehat{e}_1) + \sum_{\phi\in\Phi\setminus\{\underline{\phi}\}}\Delta_\phi\int_{y\in\mathbb{R}}\widehat{\alpha}\left(\phi\middle|y\right)\psi\left(y\middle|\widehat{e}_1\right)\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)dy\right) \\ &= \int_{y\in\mathbb{R}}\psi\left(y\middle|\widehat{e}_1\right)dy\left(\left(\underbrace{v_1(\widehat{e}_1)+\lambda u_1'(\widehat{e}_1)}_{\text{independent of } \widehat{\alpha}(\cdot)}\right) + \sum_{\phi\in\Phi\setminus\{\underline{\phi}\}}\Delta_\phi\widehat{\alpha}\left(\phi\middle|y\right)\left(2-\frac{\gamma}{\beta}+\lambda\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)\right)\right).\end{aligned}$$

Maximizing the above "pointwise", i.e., at each y, it suffices to maximize the term in the brackets above. Since the first term in the brackets is independent of the control $\widehat{\alpha}(\cdot)$, it suffices to focus on the second term. Here, we have a linear program of the following form:

$$\begin{aligned}&\sum_{\phi\in\Phi\setminus\{\underline{\phi}\}}\widehat{\alpha}(\phi|y)\Delta_\phi\left(2-\frac{\gamma}{\beta}+\lambda\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)\right)\\ \text{s.t.}\quad &\sum_{\phi\in\Phi\setminus\{\underline{\phi}\}}\widehat{\alpha}(\phi|y)\le 1, \widehat{\alpha}(\phi|y)\in[0,1].\end{aligned}$$

Since $0\le\Delta_\phi<\Delta_{\overline{\phi}}$ for all $\phi\in\Phi\setminus\{\overline{\phi}\}$, it follows that:

$$\begin{aligned}\widehat{\alpha}\left(\overline{\phi}\middle|y\right) &= \begin{cases}1, \text{ if } 2-\frac{\gamma}{\beta}+\lambda\left(\frac{y-\widehat{e}_1}{\sigma^2}\right)>0;\\ 0, \text{ o/w.}\end{cases}\\ \widehat{\alpha}\left(\phi\middle|y\right) &= 0 \text{ for all } \phi\in\Phi\setminus\{\overline{\phi},\underline{\phi}\}.\end{aligned}\tag{A.40}$$

Using a similar approach as in Theorem 3 – substituting (A.40) in the first-order condition above – we can show that for any $\widehat{e}_1\ge e_A$, we have that $\lambda>0$. Therefore, a "threshold" lottery, that places a mass of 1 on resource $\overline{\phi}$ if $y>\widehat{e}_1-\frac{\sigma^2}{\lambda}\left(2-\frac{\gamma}{\beta}\right)$ and a mass of 1 on resource $\underline{\phi}$ otherwise, is optimal. Q.E.D.

A.5. Proofs of Results in Section 9

Proof of Theorem 8: We write Problem AS – Uniform as follows:

$$\begin{aligned}\max_{\widehat{\alpha}(\cdot),\{\widehat{e}_{\theta1}\}_{\theta\in\Theta}} V &= \sum_{\theta\in\Theta}f_\theta\left(v_1(\widehat{e}_{\theta1};\theta)+\left(2-\frac{\gamma}{\beta}\right)\Delta_\theta\int_{y\in\mathbb{R}}\widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_{\theta1}\right)dy\right)\\ \text{s.t. } \widehat{e}_{\theta1} &\in \arg\max_{e_1\ge0}\left(u_{\theta1}(e_1)+\Delta_\theta\int_{y\in\mathbb{R}}\widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_{\theta1}\right)dy\right) \text{ for each } \theta\in\Theta.\end{aligned}$$

We adopt the first-order approach as before, and replace the constraint for each agent type $\theta\in\Theta$ in the problem above with the corresponding first-order condition. We then have the following problem:

$$\begin{aligned}\max_{\widehat{\alpha}(\cdot),\{\widehat{e}_{\theta1}\}_{\theta\in\Theta}} V &= \sum_{\theta\in\Theta}f_\theta\left(v_1(\widehat{e}_{\theta1};\theta)+\left(2-\frac{\gamma}{\beta}\right)\Delta_\theta\int_{y\in\mathbb{R}}\widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_{\theta1}\right)dy\right)\\ \text{s.t.}\quad &\underbrace{1-\beta c_\theta\widehat{e}_{\theta1}}_{u_{\theta1}'(\widehat{e}_{\theta1})}+\Delta_\theta\int_{y\in\mathbb{R}}\widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_{\theta1}\right)\left(\frac{y-\widehat{e}_{\theta1}}{\sigma^2}\right)=0 \text{ for each } \theta\in\Theta.\end{aligned}$$

We write the Lagrangian as follows:

$$
\begin{aligned}
\mathcal{L} =& \sum_{\theta\in\Theta} f_\theta \left(v_1(\widehat{e}_{\theta 1};\theta) + \left(2-\frac{\gamma}{\beta}\right)\Delta_\theta \int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_{\theta 1}\right)dy \right) \; + \\
& \sum_{\theta\in\Theta} \lambda_\theta \left(1-\beta c_\theta \widehat{e}_{\theta 1} + \Delta_\theta \int_{y\in\mathbb{R}} \widehat{\alpha}(y)\psi\left(y\middle|\widehat{e}_{\theta 1}\right)\left(\frac{y-\widehat{e}_{\theta 1}}{\sigma^2}\right)\right) \\
=& \sum_{\theta\in\Theta} \int_{y\in\mathbb{R}} \psi\left(y\middle|\widehat{e}_{\theta 1}\right)dy \left[\left(f_\theta v_1(\widehat{e}_{\theta 1};\theta) + \lambda_\theta\left(1-\beta c_\theta \widehat{e}_{\theta 1}\right)\right) + \widehat{\alpha}(y)\left(\Delta_\theta\left(f_\theta\left(2-\frac{\gamma}{\beta}\right)+\lambda_\theta\left(\frac{y-\widehat{e}_{\theta 1}}{\sigma^2}\right)\right)\right)\right] \\
=& \underbrace{\int_{y\in\mathbb{R}} \left(\sum_{\theta\in\Theta}\psi\left(y\middle|\widehat{e}_{\theta 1}\right)\left(f_\theta v_1(\widehat{e}_{\theta 1};\theta)+\lambda_\theta\left(1-\beta c_\theta\widehat{e}_{\theta 1}\right)\right)\right)dy}_{\text{independent of } \widehat{\alpha}(y)} + \\
& \int_{y\in\mathbb{R}} \left(\underbrace{\sum_{\theta\in\Theta}\psi\left(y\middle|\widehat{e}_{\theta 1}\right)\Delta_\theta\left(f_\theta\left(2-\frac{\gamma}{\beta}\right)+\lambda_\theta\left(\frac{y-\widehat{e}_{\theta 1}}{\sigma^2}\right)\right)}_{\star} \right)\widehat{\alpha}(y)dy.
\end{aligned}
$$

It suffices to focus on the second term. It follows that

$$
\widehat{\alpha}(y) = \begin{cases} 1, \text{ if } \star \geq 0; \\ 0, \text{ if } \star < 0, \end{cases}
$$

where

$$
\star = \sum_{\theta\in\Theta} f_\theta \Delta_\theta \psi_0\left(\frac{y-\widehat{e}_{\theta 1}}{\sigma}\right)\left(2-\frac{\gamma}{\beta}+\frac{\lambda_\theta}{f_\theta\sigma}\left(\frac{y-\widehat{e}_{\theta 1}}{\sigma}\right)\right).
$$

Below, we analyze $\star$.

Let $\widehat{y} = \frac{y-\widehat{e}_{L1}}{\sigma}$, and $\delta_{\widehat{e}} = \frac{\widehat{e}_{L1}-\widehat{e}_{H1}}{\sigma}$. Then, we can write $\star$ as follows:

$$
\star = f_L\Delta_L\psi_0(\widehat{y})\left(2-\frac{\gamma}{\beta}+\frac{\lambda_L}{f_L\sigma}\widehat{y}\right) + f_H\Delta_H\psi_0(\widehat{y}+\delta_{\widehat{e}})\left(2-\frac{\gamma}{\beta}+\frac{\lambda_H}{f_H\sigma}(\widehat{y}+\delta_{\widehat{e}})\right).
$$

From Lemma B.4, it follows that if $\lambda_L, \lambda_H > 0$, $\star$ single-crosses from below. That is, there exists a unique solution to y at which $\star = 0$, and to the left (resp., right) of this point, $\star$ is negative (resp., positive). Q.E.D.

Appendix B: Helpful Results

Below, we state and prove some results that are useful for the proofs of our main results in the paper.

B.1. Useful Results on Gaussian Random Variables

LEMMA B.1. *Suppose $X \sim \mathcal{N}(\mu, \sigma^2)$. Consider any function $f : \mathbb{R} \mapsto [0,1]$. Then,*

$$-\frac{\sigma}{\sqrt{2\pi}} \leq \mathbb{E}[f(X)(X-\mu)] \leq \frac{\sigma}{\sqrt{2\pi}}.$$

Proof: Since $f(x) \in [0,1]$ for any $x \in \mathbb{R}$, for any random variable X, it follows that:

$$\mathbb{E}[(X-\mu)^-] \leq \mathbb{E}[f(X)(X-\mu)] \leq \mathbb{E}[(X-\mu)^+].$$

Since $X \sim \mathcal{N}(\mu, \sigma^2)$, we have:

$$-\frac{\sigma}{\sqrt{2\pi}} \leq \mathbb{E}[f(X)(X-\mu)] \leq \frac{\sigma}{\sqrt{2\pi}}.$$

Q.E.D.

LEMMA B.2. *Suppose $X \sim \mathcal{N}(0,1)$. Then,*

$$-1 \leq \psi_0'(x) \leq 1$$

Proof: Observe that

$$\psi_0'(x) = -\psi_0(x)x.$$

Note that $\psi_0'(-x) = -\psi_0'(x)$. Also, $\psi_0'(x)$ is positive iff $x < 0$. Further, $\psi_0'(x)$ is quasiconcave. Hence, the first-order-condition suffices. Using the first-order-condition, we have:

$$\psi_0''(x) = 0 \implies x = \pm 1.$$

Substituting the above, we have the required result.

Q.E.D.

B.2. Some Useful Functions and Their Properties

Recall the definition of $\widehat{g}(z)$ from (A.36):

$$\widehat{g}(z) = z(\psi_{0(-)}^{-1})'(z).$$

LEMMA B.3. *$\widehat{g}(z)$ is increasing in z.*

Proof: Since $\psi_{0(-)}^{-1}(z) = -\sqrt{2\log\left(\frac{1}{\sqrt{2\pi}z}\right)}$, we have:

$$\begin{aligned} \widehat{g}'(z) &= -z\left(\sqrt{2\log\left(\frac{1}{\sqrt{2\pi}z}\right)}\right)' \\ &= \frac{1}{\sqrt{2\log\left(\frac{1}{\sqrt{2\pi}z}\right)}}\,(>0)\,. \end{aligned}$$

Hence, $\widehat{g}(\cdot)$ is increasing.

Q.E.D.

Consider any positive numbers a_i, b_i for $i \in \{1,2\}$. Let $\widehat{r}_i(z)$ denote the following function:

$$\widehat{r}_i(z) = (a_i + b_i z)\,\psi_0(z).$$

Consider a positive number ϵ. Let $R(z)$ denote the following:

$$\begin{aligned} R(z) &= r_1(z) + r_2(z+\epsilon) \\ &= \psi_0(z)\left(\underbrace{(a_1+b_1 z) + (a_2 + b_2(z+\epsilon))\,\frac{\psi_0(z+\epsilon)}{\psi_0(z)}}_{\triangleq \rho(z)}\right). \end{aligned}$$

Lemma B.4. *$R(z)$ is single-crossing in z from below.*

Proof: First, observe that $\widehat{r}_i(z)$ single-crosses 0 from below, with a unique root at $z = -\frac{a_i}{b_i}$. Next, one of two cases arises:

(i) $-\left(\frac{a_2}{b_2} + \epsilon\right) \leq -\frac{a_1}{b_1}$.

(ii) $-\frac{a_1}{b_1} < -\left(\frac{a_2}{b_2} + \epsilon\right)$.

Consider part (i). Observe that for $i \in \{1,2\}$, $r_i(z) < 0$ if $z \leq -\left(\frac{a_2}{b_2} + \epsilon\right)$, and $r_i(z) > 0$ if $z \geq -\frac{a_1}{b_1}$. Consider some $\widehat{z} \in \left(-\left(\frac{a_2}{b_2} + \epsilon\right), -\frac{a_1}{b_1}\right)$. Suppose that at $z = \widehat{z}$, $R(\widehat{z}) > 0$. We will show that for any $\delta > 0$, $R(\widehat{z} + \delta) > 0$. Since $R(\widehat{z}) > 0$, it must be the case that $\rho(\widehat{z}) > 0$. We write $R(\widehat{z}+\delta)$ as follows:

$$\begin{aligned} R(\widehat{z}+\delta) &= r_1(\widehat{z}+\delta) + r_2(\widehat{z}+\delta+\epsilon) \\ &= \psi_0(\widehat{z}+\delta)\rho(\widehat{z}+\delta) \\ &= \psi_0\left(\widehat{z}+\delta\right)\left(\underbrace{\rho(\widehat{z}) + b_1\delta + b_2\delta\frac{\psi_0(\widehat{z}+\delta+\epsilon)}{\psi_0(\widehat{z}+\delta)} + (a_2 + b_2(\widehat{z}+\epsilon))\left(1 - e^{\delta\epsilon}\right)e^{-\epsilon(\widehat{z}+\delta+\frac{\epsilon}{2})}}_{\rho(\widehat{z}+\delta)}\right). \end{aligned}$$

The right-hand-side is positive, since each term inside the brackets is positive; hence $R(\widehat{z}+\delta) > 0$. An identical approach applies to (ii).

Q.E.D.

Appendix C: A Generalization of the Model in Section 3

We consider the following more-general version of the problem described in the main paper. Let $y_1 \in \mathcal{Y} \subseteq \mathbb{R}$ where $\mathcal{Y}$ is compact. Let the p.d.f. and c.d.f of $y_1|e_1$ be the following:

$$y_1 \Big| e_1 \sim \psi\left(\cdot \Big| e_1\right), \Psi\left(\cdot \Big| e_1\right).$$

We assume that $y_1|e_1$ satisfies the strict Monotone Likelihood Ratio Property (MLRP). That is, let

$$l\left(y \Big| e\right) = \frac{\psi_e\left(y \Big| e\right)}{\psi\left(y \Big| e\right)}.$$

We assume that $l(y|e)$ is strictly increasing in y.

Let $\tilde{v}_t(y_t, e_t)$ and $\tilde{u}(y_t, e_t)$ denote, respectively, the period-t payoff of the principal and the agent from an outcome y_t and agent effort e_t. Let $v_1(e_1)$ and $u_1(e_1)$ denote, respectively, denote the principal's and the agent's expected period-1 payoff from an agent effort e_1. That is,

$$v_1(e_1) = \mathbb{E}_{y_1|e_1}\left[\tilde{v}_1(y_1, e_1)\right] \text{ and } u_1(e_1) = \mathbb{E}_{y_1|e_1}\left[\tilde{u}_1(y_1, e_1)\right].$$

We assume that $v_1(\cdot)$ and $u_1(\cdot)$ are single peaked, and $u_1(\cdot)$ is strictly concave. Let e_P and e_A denote, respectively, the preferred period-1 efforts of the principal and agent. We assume $e_P > e_A$.

Let Φ denote the finite set of resources available to the principal. In period 2, let $e_2^\star(\phi)$ denote the agent's optimal effort if the agent is provided with resource $\phi \in \Phi$. That is,

$$e_2^\star(\phi) = \arg\max_{e_2 \geq 0} \mathbb{E}_{y_2|e_2}\left[\tilde{u}_2(y_2, e_2, \phi)\right].$$

If the right-hand-side is not singleton, then we let $e_2^\star(\phi)$ denote the principal's preferred effort. Let

$$v_2(\phi) = \mathbb{E}_{y_2|e_2}[\tilde{v}(y_2, e_2^\star(\phi), \phi)] \text{ and } u_2(\phi) = \mathbb{E}_{y_2|e_2}[\tilde{u}(y_2, e_2^\star(\phi), \phi)].$$

We assume that the principal and the agent have identical preferences over Φ in the following manner. Consider $\phi, \phi' \in \Phi$:

$$u_2(\phi) \geq u_2(\phi') \Leftrightarrow v_2(\phi) \geq v_2(\phi').$$

Consequently, we say that $\phi \succ \phi'$ (ϕ is "preferred" or "superior" to ϕ') if the principal and the agent's period-2 expected payoffs under ϕ are larger than under ϕ'. Let $\overline{\phi}$ and $\underline{\phi}$ denote the most-preferred and least-preferred resource. Without loss of generality, we assume that for $\phi \neq \phi'$, either $u_2(\phi) \neq u_2(\phi')$ or $v_2(\phi) \neq v_2(\phi')$, or both. For any resource $\phi \in \Phi$, define the strict upper (resp., lower) resource set following:

$$\begin{aligned} \mathsf{SURS}(\phi) &= \{\phi' \in \Phi : \phi' \succ \phi\}. \\ \mathsf{SLRS}(\phi) &= \{\phi' \in \Phi : \phi \succ \phi'\}. \end{aligned}$$

Indeed, $\mathsf{SURS}(\overline{\phi}) = \mathsf{SLRS}(\underline{\phi}) = \emptyset$. For any resource ϕ and any $\phi' \in \mathsf{SLRS}(\phi)$, define $r(\phi||\phi')$ as follows:

$$r\left(\phi||\phi'\right) = \frac{v_2(\phi) - v_2(\phi')}{u_2(\phi) - u_2(\phi')}.$$

The ratio $r(\phi||\phi')$ measures the preference of the resource ϕ to ϕ' for the principal relative to the agent. The principal's problem can be written as follows:

$$\max_{\{\widehat{\alpha}(\phi|\cdot)\}_{\phi\in\Phi},\widehat{e}_1} V = v_1(\widehat{e}_1) + \sum_{\phi\in\Phi} v_2(\phi) \int_{y\in\mathbb{R}} \widehat{\alpha}\left(\phi\middle|y\right)\psi\left(y\middle|\widehat{e}_1\right)dy$$

$$\widehat{e}_1 \in \arg\max_{e_1\geq 0}\; u_1(e_1) + \sum_{\phi\in\Phi} u_2(\phi) \int_{y\in\mathbb{R}} \widehat{\alpha}\left(\phi\middle|y\right)\psi\left(y\middle|e_1\right)dy.$$

Fix an effort, say $\widehat{e}_1$, $\widehat{e}_1 \geq e_A$ that can be induced.[1] We identify an optimal contract that induces $\widehat{e}_1$.

As before, we adopt the first-order approach, i.e., we replace the constraint with the agent's first-order condition. We have the following problem, denoted by PROBLEM $\mathbf{P}(\widehat{e}_1)$:

$$\left.\begin{aligned} &\max_{\{\widehat{\alpha}(\phi|\cdot)\}_{\phi\in\Phi}} V = v_1(\widehat{e}_1) + \sum_{\phi\in\Phi} v_2(\phi) \int_{y\in\mathbb{R}} \widehat{\alpha}\left(\phi\middle|y\right)\psi\left(y\middle|\widehat{e}_1\right)dy \\ &\text{s.t. } u_1'(\widehat{e}_1) + \sum_{\phi\in\Phi} u_2(\phi) \int_{y\in\mathbb{R}} \widehat{\alpha}\left(\phi\middle|y\right)\psi_e\left(y\middle|\widehat{e}_1\right)dy = 0. \end{aligned}\right\} \quad (\text{PROBLEM } \mathbf{P}(\widehat{e}_1))$$

The Lagrangian can be written as follows:

$$\mathcal{L} = \int_{y\in\mathbb{R}} \psi\left(y\middle|\widehat{e}_1\right)\left((v_1(\widehat{e}_1) + \lambda u_1'(\widehat{e}_1)) + \underbrace{\sum_{\phi\in\Phi}\widehat{\alpha}\left(\phi\middle|y_1\right)\left(v_2(\phi) + \lambda u_2(\phi) l\left(y_1\middle|\widehat{e}_1\right)\right)}_{\star}\right)dy.$$

We maximize the term inside the brackets in the right-hand-side "pointwise". It suffices to focus on the term denoted by $\star$. This results in following linear program:

$$\begin{aligned} \max_{\{\widehat{\alpha}(\cdot|y_1)\}_{\phi\in\Phi}} \quad & \sum_{\phi\in\Phi}\widehat{\alpha}\left(\phi\middle|y_1\right)\left(v_2(\phi) + \lambda u_2(\phi) l\left(y_1\middle|\widehat{e}_1\right)\right) \\ \text{s.t} \quad & \sum_{\phi\in\Phi}\widehat{\alpha}\left(\phi\middle|y_1\right) = 1, \widehat{\alpha}\left(\phi\middle|y_1\right) \in [0,1]. \end{aligned}$$

We can show that, at optimality, $\lambda > 0$. We have the following result.

THEOREM C.1. *At any y_1, a degenerate lottery arises. That is, for any y_1 and any $\phi \in \Phi$, $\widehat{\alpha}(\phi|y_1) \in \{0,1\}$.*

Proof: The problem above is a linear program over the probability simplex of the $[0,1]^{|\Phi|}$ hypercube. Thus, there exists an optimal solution at a corner point and any such solution corresponding to a degenerate lottery.

Q.E.D.

The above result states that, for any given output, there exists an optimal solution in which there is no mixing of resources.

[1] The set of efforts that can be induced is bounded.

C.1. Two Types of Resource ($|\Phi| = 2$)

Suppose $|\Phi| = 2$, i.e., $\Phi = \{\underline{\phi}, \overline{\phi}\}$. We have the following result.

THEOREM C.2. *If* $|\Phi| = 2$*, then*

$$\widehat{\alpha}\left(\overline{\phi}\middle|y\right) = \begin{cases} 1, \text{ if } y \geq l^{-1}\left(-\frac{r(\overline{\phi}||\underline{\phi})}{\lambda}\middle|\widehat{e}_1\right); \\ 0, \ o/w. \end{cases}$$

Proof: Resource $\overline{\phi}$ is provided w.p. 1 iff following condition holds:

$$\begin{aligned} v_2(\overline{\phi}) + \lambda u_2(\overline{\phi}) l\left(y_1\middle|\widehat{e}_1\right) &\geq v_2(\underline{\phi}) + \lambda u_2(\underline{\phi}) l\left(y_1\middle|\widehat{e}_1\right) \\ \implies l\left(y_1\middle|\widehat{e}_1\right) &\geq -\frac{1}{\lambda} r(\overline{\phi}||\underline{\phi}). \end{aligned}$$

Since $l(\cdot|\widehat{e}_1)$ is strictly increasing, we have the claimed result. Q.E.D.

Thus, if $|\Phi| = 2$, to induce any effort $\widehat{e}_1$, an optimal contract is a threshold contract. Consequently, in equilibrium, the principal uses a threshold contract. Indeed, in Section 5, $l(y|e) = \frac{y-e}{\sigma^2}$ and $r(\overline{\phi}||\underline{\phi}) = 2 - \frac{\gamma}{\beta}$. Hence, the threshold condition simplifies to $y \geq e_1 - \frac{\sigma^2}{\lambda}\left(2 - \frac{\gamma}{\beta}\right)$ as shown in (A.32).

To solve for λ (for a given $\widehat{e}_1$), let $\Delta_u = u_2(\overline{\phi}) - u_2(\underline{\phi})$. Substituting the above solution in the agent's first-order condition, we have:

$$\int_{y=l^{-1}\left(-\frac{r(\overline{\phi}||\underline{\phi})}{\lambda}\middle|\widehat{e}_1\right)}^{\infty} \psi_e\left(y\middle|\widehat{e}_1\right) dy = -\frac{u_1'(\widehat{e}_1)}{\Delta_u}. \qquad \text{(solution to } \lambda \text{ for given } \widehat{e}_1)$$

Further, one can confirm from the above that $\lambda > 0$ for any $\widehat{e}_1 > e_A$ since $u_1'(\widehat{e}_1) < 0$ for $\widehat{e}_1 > e_A$.

Let $\Delta_v = v_2(\overline{\phi}) - v_2(\underline{\phi})$. The principal's optimization problem simplifies to the following single-dimensional optimization problem:

$$\begin{aligned} &\max_{\widehat{e}_1} V = v_1(\widehat{e}_1) + \Delta_v\left(1 - \Psi\left(\tau(\widehat{e}_1)\middle|\widehat{e}_1\right)\right) \\ &\text{where} \qquad \int_{y=\tau(\widehat{e}_1)}^{\infty} \psi_e\left(y\middle|\widehat{e}_1\right) dy = -\frac{u_1'(\widehat{e}_1)}{\Delta_u}. \end{aligned}$$

For any $\widehat{e}_1$, the constraint above identifies $\tau(\widehat{e}_1)$. The problem above identifies the optimal induced effort.

C.2. Multiple Types of Resources ($|\Phi| \geq 3$)

Suppose $|\Phi| \geq 3$, i.e., $\{\underline{\phi}, \overline{\phi}\} \subsetneq \Phi$. For any $\widehat{e}_1$, define the following:

$$\underline{y}_{\phi}(\widehat{e}_1) = l^{-1}\left(-\frac{1}{\lambda}\min_{\phi' \in \mathsf{SLRS}(\phi)} r(\phi||\phi')\middle|\widehat{e}_1\right) \quad \text{and} \tag{C.41}$$

$$\overline{y}_{\phi}(\widehat{e}_1) = l^{-1}\left(-\frac{1}{\lambda}\max_{\phi' \in \mathsf{SURS}(\phi)} r(\phi'||\phi)\middle|\widehat{e}_1\right). \tag{C.42}$$

Since $\mathsf{SURS}(\overline{\phi}) = \mathsf{SLRS}(\underline{\phi}) = \emptyset$, we let $\underline{y}_{\underline{\phi}}(\widehat{e}_1) = -\infty$ and $\overline{y}_{\overline{\phi}}(\widehat{e}_1) = \infty$. Observe that $\underline{y}_{\phi}(\widehat{e}_1)$ (resp., $\overline{y}_{\phi}(\widehat{e}_1)$) is finite for any $\phi \neq \underline{\phi}$ (resp., $\phi \neq \overline{\phi}$). Let $\mathcal{Y}_{\phi}(\widehat{e}_1)$ denote the following:

$$\mathcal{Y}_{\phi}(\widehat{e}_1) = (\underline{y}_{\phi}(\widehat{e}_1), \overline{y}_{\phi}(\widehat{e}_1)].$$

The sets $\mathcal{Y}_{\overline{\phi}}(\widehat{e}_1)$ and $\mathcal{Y}_{\underline{\phi}}(\widehat{e}_1)$ are non-empty. For $\phi \notin \{\underline{\phi}, \overline{\phi}\}$, the set $\mathcal{Y}_{\phi}(\widehat{e}_1)$ is non-empty iff the following holds:

$$\max_{\phi' \in \mathsf{SURS}(\phi)} r(\phi' || \phi) < \min_{\phi' \in \mathsf{SLRS}(\phi)} r(\phi || \phi'). \tag{C.43}$$

Define the following:

$$\Phi^{\mathsf{relevant}} = \{\overline{\phi}, \underline{\phi}\} \cup \{\phi : \phi \text{ satisfies (C.43)}\}$$

Observe that (C.43) does not depend on $\widehat{e}_1$. Therefore, a resource $\phi \in \Phi^{\mathsf{relevant}}$ is relevant for any e_1, and hence for the equilibrium effort e_1. Loosely, condition (C.43) holds for resource ϕ if the principal has a "stronger preference" for resource ϕ relative to the agent. Let $\Phi^{\mathsf{irrelevant}} = \Phi \setminus \Phi^{\mathsf{relevant}}$. We have the following result:

Theorem C.3. *If $|\Phi| \geq 3$, then,*

$$\widehat{\alpha}\left(\phi \middle| y_1\right) = \begin{cases} 1, & \text{if } y \in \mathcal{Y}_{\phi}(\widehat{e}_1); \\ 0, & o/w. \end{cases}$$

Proof: From Theorem C.1, we have that at any y_1, there exists ϕ s.t. $\widehat{\alpha}(\phi|y_1) = 1$, and for all $\phi' \in \Phi \setminus \{\phi\}$, $\widehat{\alpha}(\phi'|y_1) = 0$. For a resource ϕ to be provided, we require that:

$$\begin{aligned} v_2(\phi) + \lambda u_2(\phi) l\left(y \middle| \widehat{e}_1\right) \quad &> \quad \max_{\phi' \in \Phi \setminus \{\phi\}} v_2(\phi) + \lambda u_2(\phi) l\left(y \middle| \widehat{e}_1\right) \\ \implies (v_\phi - v_{\phi'}) + \lambda l\left(y \middle| \widehat{e}_1\right)(u_\phi - u_{\phi'}) \quad &> \quad 0 \text{ for each } \phi \in \Phi \\ \implies l\left(y \middle| \widehat{e}_1\right) < -\frac{r(\phi' || \phi)}{\lambda} \text{ for } \phi' \in \mathsf{SURS}(\phi) \;\; &\text{and} \;\; l\left(y \middle| \widehat{e}_1\right) > -\frac{r(\phi || \phi')}{\lambda} \text{ for } \phi' \in \mathsf{SLRS}(\phi). \end{aligned}$$

Since $l(\cdot|\widehat{e}_1)$ is strictly increasing, we get the claimed result. Q.E.D.

In particular, observe that for any $\phi \in \Phi^{\mathsf{irrelevant}}$, we have that $\widehat{\alpha}(\phi|y_1) = 0$ for all y_1, i.e., the resource ϕ is never provided. Further, the set of output levels $\mathcal{Y}_{\phi}(\widehat{e}_1)$ where resource ϕ is provided is a contiguous set (i.e., no holes). Also, for any ϕ, ϕ' that are relevant and any $\widehat{e}_1$, $\mathcal{Y}_{\phi}(\widehat{e}_1) \cap \mathcal{Y}_{\phi'}(\widehat{e}_1) = \emptyset$.

In our analysis in Section 7, observe that $r(\phi||\phi') = 2 - \frac{\gamma}{\beta}$. Hence, $\Phi^{\mathsf{relevant}} = \{\underline{\phi}, \overline{\phi}\}$. Indeed, if $r(\phi||\phi')$ is a constant for all $\phi' \in \mathsf{SLRS}(\phi)$, $\phi \in \Phi \setminus \{\underline{\phi}\}$, then, $\Phi^{\mathsf{relevant}} = \{\underline{\phi}, \overline{\phi}\}$.

Lemma C.1. *Consider two resources $\phi, \phi' \notin \{\underline{\phi}, \overline{\phi}\}$ and any inducible effort $\widehat{e}_1$. Suppose $\phi' \succ \phi$. Then,*

$$\overline{y}_{\phi}(\widehat{e}_1) \leq \underline{y}_{\phi'}(\widehat{e}_1). \tag{C.44}$$

Proof: Consider two resources ϕ, ϕ' where $\phi' \succ \phi$ and an inducible effort $\widehat{e}_1$. By definition, the following holds:

$$\min_{\widehat{\phi} \in \mathsf{SLRS}(\phi')} r(\phi' || \widehat{\phi}) \leq r(\phi' || \phi) \leq \max_{\widehat{\phi} \in \mathsf{SURS}(\phi)} r(\widehat{\phi} || \phi).$$

Recall Problem $\mathbf{P}(\widehat{e}_1)$, the principal's problem that identifies the resource-allocation strategy to induce $\widehat{e}_1$. In Problem $\mathbf{P}(\widehat{e}_1)$, $\lambda > 0$ at optimality. Therefore, from above,

$$-\frac{1}{\lambda} \min_{\widehat{\phi} \in \mathsf{SLRS}(\phi')} r(\phi' || \widehat{\phi}) \geq -\frac{1}{\lambda} \max_{\widehat{\phi} \in \mathsf{SURS}(\phi)} r(\widehat{\phi} || \phi).$$

Recall that $l(y|e)$ is increasing in y for any e (by assumption). Therefore, we have:

$$l^{-1}\left(-\frac{1}{\lambda} \min_{\widehat{\phi} \in \mathsf{SLRS}(\phi')} r(\phi' || \widehat{\phi}) \middle| \widehat{e}_1\right) \geq l^{-1}\left(-\frac{1}{\lambda} \max_{\widehat{\phi} \in \mathsf{SURS}(\phi)} r(\widehat{\phi} || \phi) \middle| \widehat{e}_1\right).$$

From (C.41) and (C.42), the above condition is the same as $\underline{y}_{\phi'}(\widehat{e}_1) \geq \overline{y}_{\phi}(\widehat{e}_1)$, which is the required inequality. Q.E.D.

The implication from the result above is that the resource allotted is "monotone" in y_1. That is, a higher period-1 output results in a (weakly) superior resource.