“Seemingly-Beneficial” Interventions

(Forthcoming, Production and Operations Management)

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Organizations routinely introduce hitherto-unexplored interventions to improve their supply chains. Consider a principal (e.g., a firm) that implements a “seemingly-helpful” intervention: For any *fixed* actions of the principal and the agents (e.g., consumers), the principal’s payoff is higher in the presence of the intervention than in its absence. While one would expect such well-intentioned interventions to benefit the principal, several papers within the OM literature show that the principal’s *equilibrium* payoff can be hurt, even ignoring the intervention’s implementation cost. While this conclusion is often based on analyzing a single-shot, simultaneous-move game, repeated-interactions can also serve as an appropriate environment in many cases. A fundamental question arises: Does this conclusion hold even under repeated interactions?

We study this question using the framework of infinitely-repeated games and the notion of a precommitment equilibrium from the literature on reputation in repeated games. We identify two key characteristics that determine whether a seemingly-beneficial intervention helps, or can possibly hurt the firm: (*i*) *nature of the intervention* (ceteris paribus, does it induce agents to react in a manner favorable to the principal?), and (*ii*) *extent of interaction* (single-shot at one extreme and infinitely-repeated at the other). Interestingly, we demonstrate the following two possibilities using settings analyzed in the recent OM literature: seemingly-beneficial interventions can (a) hurt the firm in a single-shot analysis but benefit under repeated interactions, and (b) continue to hurt the firm under repeated interactions. We also obtain easy-to-interpret conditions under which the benefit of such interventions is guaranteed under repeated interactions.

Key words: Seemingly-Beneficial Interventions, Repeated Games, Precommitment Equilibrium.

History: Received: March, 2020. Accepted: March, 2021, by Michael Pinedo, after 1 revision.

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1. Introduction

The operations and supply chain management literature is replete with examples where researchers propose interventions that are aimed at improving supply chains by reducing the inherent frictions that make trade less efficient. These interventions could either be firm-driven, e.g., technology-driven operational interventions, or social planner-driven, e.g., governmental policy interventions. The typical goal of such interventions is to improve the welfare of one or all members of the supply chain. Examples of policy interventions by a social planner include measures to improve the accessibility and availability of goods to consumers, and promote sustainable practices. Several such interventions have been investigated in the literature: (a) governmental price-support schemes in developing economies to improve the production of foodgrains by the farmers and their consumption by the below-poverty-line population (Guda et al. 2021), (b) interventions to improve the quality of milk by reducing adulteration (Mu et al. 2014, 2016), (c) producer and consumer subsidies to incentivize the adoption of electric vehicles (Avci et al. 2014), (d) interventions to incentivize and improve the adoption of roof-top solar panels (Cohen et al. 2015), and (e) interventions to promote recycling and remanufacturing (Atasu et al. 2009). Examples of firm-driven interventions that have been examined in the recent literature include: (a) interventions introduced by service firms to reduce customer waiting and congestion through mechanisms such as mobile order-and-pay (Gao and Su 2018), (b) interventions by retailers to improve customer shopping experience, such as information about real-time in-store inventory (Aydinliyim et al. 2017) and better information-provision mechanisms in the form of physical and virtual showrooms (Gao and Su 2016), (c) interventions by software firms to protect users from software vulnerabilities, e.g., by providing monetary rewards to identifiers for each vulnerability reported (Kannan and Telang 2005) and (d) interventions by supply chain partners to reduce demand variability, e.g., through better demand-forecasting and risk-pooling through transshipment (Zhang 2005, Li and Petruzzi 2017).

Of special interest to us in this paper are settings/interventions that are "seemingly-beneficial"; that is, well-intentioned interventions that, on the surface, seem profitable to some or all interacting parties, but result in hurting them when their strategic incentives are considered. Our work focuses on such interventions. For concreteness, we begin with some examples. Mu et al. (2014, 2016) consider several settings to study adulteration in the milk supply chain in developing countries. They model the interactions between a population of farmers and a milk station (that collects milk from the farmers), and design incentives to deter adulteration of milk by farmers; such interventions/incentives can be adopted either by the social planner or the station. They find that interventions by the social planner aimed at lowering the cost of producing high-quality milk, such as providing farmers with storage and refrigeration equipment, may *not* lead to an improvement in the quality of milk. In a similar vein, Gao and Su (2016) find that an omnichannel retailer who adopts information-provision mechanisms such as physical or virtual showrooms, that provide more information about products to consumers, may lead to an *increase* in the returns rate. Key to such outcomes is the *extent* of the interaction: In both the examples above, the analysis is conducted through a single-shot, simultaneous-move game, and therefore, the short-term incentives of the players are deemed important. Nevertheless, in a variety of settings, including the ones above, repeated interactions are natural and may also serve as an appropriate setting for analysis. For instance, Mu et al. (2014) detail the interactions between the farmers and the station that occur at the start of each day over a long horizon.[1] Likewise, modeling the interactions between a retailer and a population of consumers over multiple seasons has been the kernel of many papers in OM; see Popescu and Wu (2007), Su and Zhang (2009), Liu and Van Ryzin (2011). More generally, many applications can be viewed using both lenses – as a one-time interaction, or repeated over a long horizon. Naturally, then, the following question arises: Do conclusions that arise from the analysis of a one-shot game also hold when repeated interactions are considered? This is the main question we examine in this paper.

In what follows, we present a model of a seemingly-beneficial intervention in Section 2, which encompasses the common features of beneficial interventions studied in many OM settings. We consider a fairly general setting – a principal (e.g., a firm, such as a retailer, or a government) who interacts with multiple agents (e.g., consumers, citizens) – and define a seemingly-beneficial intervention as follows: For any fixed actions of the principal and all agents, the payoff of the principal is higher in the presence of the intervention relative to that in its absence (formally defined in Assumption 2.2). We find that two aspects – the *nature of the intervention* and the *extent of the interaction* – play a key role in determining whether the intervention always benefits the principal. In particular, we identify a key property of a seemingly-beneficial intervention, viz., inducing beneficial actions from the agents (precisely defined in Assumption 2.4), under which the benefit from such interventions can be confirmed; see Theorem 2.1. Subsequently, in Section 3, we consider two recent papers: Mu et al. (2014) and Gao and Su (2016), who study seemingly-beneficial interventions in the context of the milk supply chain, and omnichannel retailing, respectively. We illustrate our key result in their respective settings. Finally, Section 4 summarizes the main managerial implications of our work.

2. Seemingly-Beneficial Interventions: A General Model

Our aim in this section is to propose a general game-theoretic model of seemingly-beneficial interventions. To this end, we begin by studying two games, denoted by G^1 and G^2. Game G^1 represents

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[1] Quoting Mu et al. (2016), *"In developing countries, most of the milk acquired by stations is collected daily from smallholder dairy farmers in rural areas . . . A typical station receives milk from about 100-200 farmers. Most of the milk collected at a station is delivered [by farmers] in the morning between 6am and 11am. Evening delivery exists in some cases, but accounts for a negligible fraction of the total milk collected."*.

the interactions in the absence of an intervention while game G^2 represents the interactions in its presence; Section 2.1 describes these two games. In Section 2.2, we demonstrate how our general model relates to two examples that will be discussed in detail in Section 3. Next, in Section 2.3, we formally define four conditions (Assumptions 2.1 – 2.4) on the players' actions and their payoffs in the two games; in particular, Assumption 2.2 states the condition under which the intervention is deemed as being seemingly beneficial. Then, in Section 2.5, we consider the case where the two games G^1 and G^2 are played repeatedly over the long horizon. In this context, we use a key result (Lemma 2.1) from the literature on reputation and repeated games – this result helps us employ the notion of a two-stage precommitment equilibrium for an infinitely-repeated game. Our main result in this section is Theorem 2.1, which shows that, under Assumptions 2.1 – 2.4, the benefit from seemingly-beneficial interventions can be confirmed under repeated interactions.

2.1. Description of the Games: Players, Actions and Payoffs

Consider two one-shot simultaneous-move games G^j, indexed by $j \in \{1,2\}$, played between a large player and a continuum of homogeneous, anonymous, small players.[2] The two games, G^1 and G^2, represent the interactions in the absence and in the presence of an intervention, respectively.

The action space of the players in games G^j, $j \in \{1,2\}$, is identical. Let $\mathcal{A}_L$ (resp., $\mathcal{A}_S$) denote the pure-action space of the large player (resp., the small players); $\mathcal{A}_L$ (resp., $\mathcal{A}_S$) is either a finite set or a compact subset of $\mathbb{R}$. We allow for the players to use mixed actions. For any set X, we let $\Delta(X)$ denote the set of probability distributions over X; a discrete probability distribution over X that places a probability $p(x)$ on $x \in X$ is denoted by $\sum_{x\in X} p(x) \circ x$. Let $\mathcal{S}_L = \Delta(\mathcal{A}_L)$ (resp., $\mathcal{S}_S = \Delta(\mathcal{A}_S)$) denote the pure- and mixed-action space of the large player (resp., an individual small player). Let $a_L \in \mathcal{S}_L$ (resp., $a_{S,i} \in \mathcal{S}_S$) denote an action (pure or mixed) of the large player (resp., an individual small player i). The actions of all the small players induce a population-action distribution over $\mathcal{A}_S$ (i.e., the proportion of players who play a particular action); let $a_{S,A} \in \mathcal{S}_S$ denote this "average" play of the small players, i.e., distribution of the small players' actions. In particular, note that if all the small players use an identical action, say $\hat{a}_S \in \mathcal{S}_S$ (i.e., $a_{S,i} = \hat{a}_S$ for all i), then their average play is also $\hat{a}_S$.

Each player's payoff depends on their own actions, the actions of the large player, and the average play of the small players. Consider game G^j, $j \in \{1,2\}$. For any action a_L of the large player and an average play $a_{S,A}$ of the small players, let $\Pi^j(a_L, a_{S,A})$ denote the payoff of the large player. Further, given a_L and $a_{S,A}$, we denote the payoff of a small player whose individual action is $a_{S,i}$ by $\pi^j(a_{S,i}, a_L, a_{S,A})$.

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[2] In Remark 2.3, we address the case of heterogeneous small players.

Before proceeding further, we provide an overview of the two examples that we will discuss in detail in Section 3 and their correspondences with the general model in this section.

2.2. Two Applications and Their Relationship to the General Model

The two examples below are drawn from the recent literature on the milk supply chain (specifically, Mu et al. 2014, 2016) and omnichannel retailing (specifically, Gao and Su 2016). Table 1 below summarizes the correspondence of the entities in these examples with those in the general model.

- **Adulteration and Testing in the Milk Supply Chain (Section 3.1):** We study a game between a milk station and a population (modeled as a continuum/mass) of milk farmers. The milk station procures milk from the farmers, mixes it, and sells it to a downstream firm. Due to high testing costs, the station is unable to test each farmer, creating an incentive for farmers to adulterate milk. Each farmer chooses the quality of milk he produces – he chooses to produce either high or low quality milk. The station chooses its testing strategy of whether to test an individual farmer or not. The farmers and the station can also use mixed strategies: An individual farmer can randomize between high and low quality milk, while the station can randomly test an individual farmer. A farmer who is tested is paid by the milk station based on the quality of his milk, while each untested farmer is paid the high-quality price. The downstream firm pays the station based on the quality of the mixed milk. In this context, we consider an intervention by a social planner who supports the farmers by supplementing them with better refrigeration and storage equipment, which reduce the marginal cost of producing high quality milk. The correspondences with our general model are noted in Table 1.[3]
- **Information Provision Mechanisms in Omnichannel Retail (Section 3.2):** Here, we consider an omnichannel retailer, who sells a single product to consumers across two channels: in-store and online. The retailer decides the stocking quantity in-store. Individual consumers decide whether to shop in-store or online; consumers can also play mixed strategies. The retailer's profit depends on the proportion of consumers who visit each channel. An individual consumer's payoff depends on his own choice (whether to shop in-store or online), the retailer's stocking quantity, and the proportion of consumers who visit the store.[4] Table 1 summarizes the correspondences with our general model. In this context, we discuss two technology-driven interventions, viz., physical and virtual showrooms, that reduce product valuation-uncertainty for the consumers.

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[3] Observe that this is a special case of the general model we consider in Section 2.1 in that the strategic interactions among the farmers are absent, i.e., an individual farmer does not impose an externality on the other farmers. Formally, the profit of an individual farmer is independent of the quality of mixed milk.

[4] In this example, consumers impose a negative externality on each other. That is, for any stocking decision of the retailer, a higher proportion of consumers who shop in-store lowers the incentives of an individual consumer to shop in-store because they experience a greater stockout risk.

General Model (Section 2)	**Adulteration in the Milk Supply Chain (Section 3.1)**	**Information Provision in Omnichannel Retailing (Section 3.2)**
Large player	Milk station	Retailer
Large player's action	Testing strategy	In-store stocking decision
Small players	Milk farmers	Consumers
Individual small player's action	Quality of milk produced by an individual farmer	Individual consumer's choice of shopping channel
Average play of the small players	Quality of mixed milk	Proportion of consumers who shop in-store (or shop online)
Beneficial intervention	Governmental interventions to reduce marginal cost of high quality milk; e.g., supplementing farmers with better storage and refrigeration equipment	Information provision mechanisms such as physical and virtual showrooms that reduce returns

Table 1 **Correspondences between the entities in the general model and the applications in Section 3**

Consider game G^j: The set of best-responses of an individual small player, corresponding to the large player's action $a_L \in \mathcal{S}_L$ and average play of the small players $a_{S,A} \in \mathcal{S}_S$ is denoted by $\mathcal{A}^j_{S,B}(a_L, a_{S,A})$. That is,

$$\mathcal{A}^j_{S,B}(a_L, a_{S,A}) = \arg\max_{a_S \in \mathcal{S}_S} \pi^j(a_S, a_L, a_{S,A}).$$

It is straightforward that if a mixed action $\alpha_S \in \mathcal{A}^j_{S,B}(a_L, a_{S,A})$, then the pure actions on which α_S places positive probability are also best-responses.

We now introduce four assumptions on the players' actions and their payoffs in game G^j, $j \in \{1,2\}$:

2.3. Comparison of the Two Games

Consider the following assumptions:

Assumption 2.1. ***(Symmetric Best-Response)*** *For any action (pure or mixed) $a_L \in \mathcal{S}_L$ of the large player, define the set of "symmetric" best responses $\hat{\mathcal{A}}^j_{S,EQ}(a_L) \subseteq \mathcal{S}_S$ as follows:*

$$\hat{\mathcal{A}}^j_{S,EQ}(a_L) = \{a_{S,A} \in \mathcal{S}_S : a_{S,A} \in \mathcal{A}^j_{S,B}(a_L, a_{S,A})\}.$$

We assume that $\hat{\mathcal{A}}^j_{S,EQ}(a_L)$ is non-empty.

In words, $\hat{\mathcal{A}}^j_{S,EQ}(\cdot)$ denotes the "symmetric" best-response correspondence of the small players, i.e., the set of "symmetric" best-responses (pure or mixed) of the homogeneous small players for a fixed action of the large player. In the analysis that follows, this assumption allows us to restrict our attention to equilibria that involve symmetric strategies for the small players. That is, all small players play the same action and the "average play best-response" of the small players is identical to the action of any individual player.[5]

In general, the set of symmetric best-responses of the small players, $\hat{\mathcal{A}}^j_{S,EQ}(a_L)$, corresponding to some action(s) of the large player, may not be a singleton. In such cases, we choose the action that leads to the lowest payoff for the large player. Formally,

$$\mathcal{A}^j_{S,EQ}(a_L) = \underset{a_{S,A} \in \hat{\mathcal{A}}^j_{S,EQ}(a_L)}{\arg\min} \Pi^j(a_L, a_{S,A}).$$

Note that $\mathcal{A}^j_{S,EQ}(a_L)$ may not be a singleton in general.

Assumption 2.2. ***(A Seemingly-Beneficial Intervention)*** *For any given actions of all the players, the large player's payoff in game G^2 is higher than that in game G^1, i.e.,*

$$\Pi^1(a_L, a_{S,A}) \leq \Pi^2(a_L, a_{S,A}) \textit{ for any } (a_L, a_{S,A})$$

Interestingly, in many applications (including those we discuss in Section 3), for fixed actions of all players, the payoff of *any* player is higher in game G^2 than in game G^1, i.e., G^2 is "seemingly-beneficial" to G^1 for all players. The focus of our paper, however, is on the impact of seemingly-beneficial interventions on the large player. Consequently, in Assumption 2.2, we only require that the intervention is seemingly-beneficial to the large player, and do not impose any such assumptions on the small players.

Assumption 2.3. ***(Increasing Average Plays)*** *Consider game G^1. Fix an action a'_L of the large player and consider two average plays $a'_{S,A}, a''_{S,A}$ of the small players. Clearly, either $\Pi^1(a'_L, a'_{S,A}) \leq \Pi^1(a'_L, a''_{S,A})$ or $\Pi^1(a'_L, a'_{S,A}) \geq \Pi^1(a'_L, a''_{S,A})$ holds. If $\Pi^1(a'_L, a'_{S,A}) \leq$ (resp., $\geq$) $\Pi^1(a'_L, a''_{S,A})$, then for any action $a_L \in \mathcal{A}_L$, we assume that $\Pi^1(a_L, a'_{S,A}) \leq$ (resp., $\geq$) $\Pi^1(a_L, a''_{S,A})$.*

In words, the large player's preferences over the small players' average play is independent of his action. Stated differently, from the perspective of the large player's payoff, there is an ordering over the elements of $\mathcal{S}_S$.

We define the following two relationships:

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[5] If $\mathcal{A}_S \subseteq \mathbb{R}$, then the symmetric best-response of the small players comprises of a distribution over $\mathcal{A}_S$. From the law of large numbers, their average action is equal to the mean of this distribution.

1. **(Comparing Two Actions of the Small Players)** Consider two average plays $a'_{S,A}$, $a''_{S,A} \in \mathcal{S}_S$. We say that an average play $a'_{S,A}$ is "smaller" (resp., "larger") than an average play $a''_{S,A}$, denoted by $a'_{S,A} \preccurlyeq$ (resp., $\succcurlyeq$) $a''_{S,A}$ if, for any (and by the above assumption, all) $a_L \in \mathcal{A}_L$, we have that $\Pi^1(a_L, a'_{S,A}) \leq \Pi^1(a_L, a''_{S,A})$. That is,
$$a'_{S,A} \preccurlyeq a''_{S,A} \Longleftrightarrow \Pi^1(a_L, a'_{S,A}) \leq \Pi^1(a_L, a''_{S,A}) \text{ for all } a_L \in \mathcal{S}_L.$$
Further, $a'_{S,A} \prec$ (resp., $\succ$) $a''_{S,A}$ if $a'_{S,A} \preccurlyeq$ (resp., $\succcurlyeq$) $a''_{S,A}$ and there exists $a_L \in \mathcal{S}_L$ such that $\Pi^1(a_L, a'_{S,A}) <$ (resp., $>$) $\Pi^1(a_L, a''_{S,A})$.
2. **(Comparing Two Sets of Actions of the Small Players)** Consider two sets of average plays of the small players, $\mathcal{A}', \mathcal{A}'' \subseteq \mathcal{S}_S$. The set of average plays $\mathcal{A}'$ is "smaller" (resp., "larger") than the set of average plays $\mathcal{A}''$, denoted by $\mathcal{A}' \preccurlyeq$ (resp., $\succcurlyeq$) $\mathcal{A}''$, if, for any $a'_{S,A} \in \mathcal{A}'$, $a''_{S,A} \in \mathcal{A}''$, we have that $a'_{S,A} \preccurlyeq$ (resp., $\succcurlyeq$) $a''_{S,A}$. Further, $\mathcal{A}' \prec$ (resp., $\succ$) $\mathcal{A}''$ if $\mathcal{A}' \preccurlyeq$ (resp., $\succcurlyeq$) $\mathcal{A}''$ and there exist $a'_{S,A} \in \mathcal{A}'$, $a''_{S,A} \in \mathcal{A}''$ such that $a'_{S,A} \prec$ (resp., $\succ$) $a''_{S,A}$.

Remark 2.1. **(Special Case: $|\mathcal{A}_S| = 2$)** If the small player's action space $\mathcal{A}_S$ consists of two actions, then the average play of the small players can be summarized with one parameter – the proportion of small players who play, say, the first action. For instance, consider the example in Section 3.2: Here, $\mathcal{A}_S = \{\texttt{In-Store}, \texttt{Online}\}$: A small player's (i.e., an individual consumer's) actions can be either shopping in-store or online. Naturally, the set of strategies $\mathcal{S}_S = \Delta(\mathcal{A}_S)$ of an individual small player i can be summarized by a single parameter, say $\phi_{S,i} \in [0,1]$, which denotes his probability of choosing action `In-Store`; the average play of the small players can also be summarised by a single parameter $\phi_{S,A} \in [0,1]$, which denotes the proportion of small players who play action `In-Store`. Further, for any in-store stocking decision, the profit of the retailer is increasing in the proportion of the consumers who shop in-store. Therefore, a "higher" average play of the small players corresponds to a greater proportion of small players who choose `In-Store`, i.e., a higher value of $\phi_{S,A}$.

Consider game G^1 and an action a_L of the large player. The profit of the large player, $\Pi^1(a_L, a_{S,A})$, can be rewritten as $\Pi^1(a_L, \phi_{S,A})$, where $\phi_{S,A}$ denotes the proportion of small players who play H. Suppose that $\Pi^1(a_L, \phi_{S,A})$ is increasing in $\phi_{S,A}$ for any $a_L \in \mathcal{A}_L$. Then, the large player's payoff is increasing in the average play of the small players. ■

Assumption 2.4. ***(Interventions Induce Higher Average Plays)*** *Consider any action of the large player $a_L \in \mathcal{S}_L$. We assume that $\mathcal{A}^1_{S,EQ}(a_L) \preccurlyeq \mathcal{A}^2_{S,EQ}(a_L)$.*

In words, the above assumption states that, for any given action of the large player, the symmetric best-response of the small players is larger under the intervention (i.e., in G^2) than the symmetric best-response of the small players in the absence of the intervention (i.e., in G^1). That is, seemingly beneficial interventions induce "beneficial actions" actions – that benefit the large player – from the small players.

2.4. Seemingly-Beneficial Interventions as One-Shot Games

There exist one-shot simultaneous-move games G^1, G^2 which satisfy Assumptions 2.1–2.4 such that in equilibrium, the larger player is worse-off in G^2. That is, a seemingly-beneficial intervention that induces beneficial actions from the small players can lead to an inferior equilibrium outcome for the large player. We demonstrate an example below.

Example 2.1. **(Seemingly-Beneficial Interventions Can Lead to Inferior Outcomes)**: The actions and the payoffs, denoted in Figure 1, are as follows: The large player chooses a row (i.e., $\mathcal{A}_L = \{U, D\}$; we allow for mixed strategies) while each small player chooses a column (i.e., $\mathcal{A}_S = \{L, R\}$). Consider game G^j, $j \in \{1, 2\}$: For any $r \in \{U, D\}$ and $c \in \{L, R\}$, $\Pi^j(r|c)$ denotes the large player's payoff if the large player chooses r and all the small players choose c; likewise, $\pi^j(c|r)$ denotes an individual small player's payoff if he chooses c and the large player chooses r. We denote these payoffs and actions in Figure 1, i.e., corresponding to row r and column c, the vector in the matrix denotes $(\Pi(r|c), \pi(c|r))$.

Suppose the large player plays $a_L = x \circ U + (1-x) \circ D$ while an individual small player i plays $a_{S,i} = \phi_i \circ L + (1-\phi_i) \circ R$. Let the average play of the small players be $a_{S,A} = \phi_A \circ L + (1-\phi_A) \circ R$. Then, for any fixed actions of the players, suppose that:

(a) The large player's payoff is linear in the average play of the small players, i.e.,

$$\Pi^j(x|\phi_A) = x\Big[\phi_A \Pi^j(U|L) + (1-\phi_A)(\Pi^j(U|R))\Big] + (1-x)\Big[\phi_A \Pi^j(D|L) + (1-\phi_A)(\Pi^j(D|R))\Big].$$

(b) An individual small player's payoff is linear in the large player's action and independent of the average play of the small players, i.e.,

$$\pi^j(\phi_i|x) = \phi_i\Big[x\pi^j(L|U) + (1-x)\pi^j(L|D)\Big] + (1-\phi_i)\Big[x\pi^j(R|U) + (1-x)\pi^j(R|D)\Big].$$

It is easy to verify that game G^2 is (strictly) seemingly-beneficial to game G^1 and Assumptions 2.1–2.4 hold. However, in equilibrium, the outcome in game G^1 involves the large player playing U and all small players playing L, while the outcome in game G^2 involves the large player playing D and all small players playing R. Therefore, the payoff of all players in game G^2 is strictly less than that in game G^1, i.e., *all* players are hurt. ■

Small Player(s)

Large Player	L	R
U	4,4	0,3
D	3,0	-1,-1

Game G^1

Small Player(s)

Large Player	L	R
U	5,5	1,4
D	6,1	2,2

Game G^2

Figure 1 **Stage games G^1 and G^2. Game G^2 is seemingly-beneficial to game G^1. However, the equilibrium outcome under G^2 is inferior to that under G^1.**

The intuition behind this result is as follows: Seemingly-beneficial interventions that induce beneficial actions incentivize the small players to exert higher actions for a fixed action of the large player. However, conditional on the small players exerting a higher action, the large player may have an incentive to lower his action. Anticipating this downward deviation from the large player, the small players exert a lower action, and therefore, the resulting equilibrium outcome hurts all players. It is important to note that the above observation pertains to the equilibrium outcomes of the single-shot games G^1 and G^2. Key to such inferior outcomes is the "anticipated" action of the large player (by the small players). In a one-shot game, there is no reason for the small players to entertain the possibility that the large player will play a high action. A natural question arises: Does this conclusion hold even under repeated interactions over the long run? It is straightforward that if the small players (rationally) anticipate a high action by the large player, they will respond favorably.

In what follows, we study the value of a seemingly-beneficial intervention to the large player over the long horizon by analyzing a repeated game where the stage-game G^j is played repeatedly.

2.5. Repeated Games and Reputation

A rich body of literature in Economics has studied repeated games between a *long-lived* large player (e.g., a firm, a government, etc.) and a continuum of *anonymous* and *long-lived* small players (e.g., consumers, citizens, etc.) under a variety of settings; see, e.g., Mailath and Samuelson (2006). Public (or observable) histories of play include the large player's actions along with the average of the small players' actions while the actions of individual small players is not observed.[6] In such repeated games, it is straightforward that the small players can do no better than "myopically" optimize: Since each small player forms a negligible part of the continuum, they cannot influence the average play of the small players. Therefore, a change in her behavior does not affect the future behavior of any player.

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[6] Alternatively, the actions of individual small player are not part of the large player's history; therefore, the large player's actions in future periods depends on the average action of the small players, but not on the actions of individual small players.

A key result for such infinitely-repeated games is as follows: If the actions of the large player are perfectly observable, as the large player's discount factor, say δ, approaches 1 (i.e., when the large player is sufficiently patient), his equilibrium *normalized present value*[7] (NrPV) converges to his "Stackelberg" payoff.[8] The intuition behind this result is as follows: If the large player persistently plays his Stackelberg strategy, the small players will eventually place a high probability on the large player playing his Stackelberg strategy. It may take the large player a while to build a "reputation" of playing his Stackelberg strategy, and in general, it may be costly for him to do so. However, if he is sufficiently patient, this cost becomes negligible. This has now come to be known as the *two-stage precommitment equilibrium*. Quoting Celentani and Pesendorfer (1996):

> *"...In a repeated game our anonymity assumption implies that each small player will play a short-run best response in each period to that period's expected play since his actions do not affect his future payoffs or the public history of the game. Consequently, in a repeated game the best possible commitment for the long-run player is to [play] the Stackelberg strategy for the corresponding static game. Moreover, the long-run player only needs to convince his effectively myopic opponents that he will follow the Stackelberg strategy in the current period in order to achieve the Stackelberg payoff for that period."*

Further, this precommitment equilibrium payoff of the large player is robust to a variety of informational settings. Quoting Levine and Pesendorfer (1995):

> *"...recent theoretical literature shows that such a two-stage precommitment equilibrium is a consequence of reputation building in a repeated setting even when precommitment is impossible. This was implicit in the work of Kreps and Wilson (1982) and of Milgrom and Roberts (1982) on reputational equilibrium and was made explicit in the work of Fundenberg and Levine (1989), Fudenberg and Levine (1992). Celentani (1991), Celentani and Pesendorfer (1996), Schmidt (1993), and others have extended the scope of this result in a variety of ways."*

Within the OM literature, Su and Zhang (2009) explicitly study the value of commitment when a retailer sells to strategic consumers who experience stockout-risk. They show how commitment arises in an infinitely-repeated game, when the retailer can influence consumers' beliefs about the in-store stocking level. In their model, consumers adaptively learn and update their beliefs about the retailer's stocking decision using an exponential learning rule. The authors show that if the retailer is sufficiently patient (discount factor is sufficiently close to one), his equilibrium stocking

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[7] For any discount factor $\delta \in [0, 1)$ and any sequence of payoffs $x_1, x_2, \ldots$, the *normalized present value* is defined to be $(1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} x_t$. We use the abbreviation NrPV instead of NPV to distinguish the normalized present value from the net-present value. The two concepts are related through the identity NrPV$= (1-\delta)$NPV.

[8] The Stackelberg payoff of the large player is the most he could obtain in a single period by publicly committing to any strategy, or equivalently, his payoff in a Stackelberg game where he is the leader.

decision converges to his Stackelberg decision, and his NrPV corresponds to his Stackelberg payoff; see Section 7.1 of their paper. Using a similar approach, in Section 3.1 of our paper, we illustrate how commitment arises in equilibrium if the station is sufficiently patient and the milk farmers adaptively learn the testing strategy of the station.

Formally, let G^j_∞, $j \in \{1,2\}$ denote the infinitely-repeated game whose stage game is G^j. We evaluate payoffs using the discounting criterion, where players discount future payoffs using a discount factor δ. Denote the equilibrium payoff (NrPV) of the large player in G^j_∞ by Π^{*j}. Let Π^j_{ST} denote the Stackelberg payoff of the large player, defined as follows:

$$\Pi^j_{ST} = \max_{a_L \in \mathcal{S}_L} \left\{ \Pi^j(a_L, a_{S,A}) \text{ s.t. } a_{S,A} \in \mathcal{A}^j_{S,EQ}(a_L) \right\} \tag{2.1}$$

The action that achieves the Stackelberg payoff is called the Stackelberg action. Then, we have the following preliminary result.

Lemma 2.1. ***(Proposition 15.3.1 of Mailath and Samuelson (2006))*** *Consider G^j_∞ and a fixed $\delta \in (0,1)$. Then, there exists a finite K s.t.*

$$\Pi^{*j} \geq \delta^K \Pi^j_{ST} + (1-\delta^K)\underline{\Pi}^j,$$

where $\underline{\Pi}^j = \min_{a_L, a_{S,A}} \Pi^j(a_L, a_{S,A})$.

Consequently, we have the following result.

Corollary 2.1. *Consider G^j_∞ and a fixed $\epsilon > 0$. There exists $\delta_\epsilon \in (0,1)$ s.t. for all $\delta \in (\delta_\epsilon, 1)$, the equilibrium NrPV of the large player exceeds $\Pi^j_{ST} - \epsilon$.*

The proof of Lemma 2.1 from the literature on reputation in repeated games adopts an "adverse selection" approach to reputation; see Chapter 15 in Mailath and Samuelson (2006). Consider an arbitrarily small perturbation of game G^j, where the large player could be one of the following "types": a *strategic* type, or a *commitment* type. The payoff of the strategic-type large player is the same as that in the stage game G^j. The commitment-type large player plays a fixed action in any period.[9] The strategic-type large player can mimic the Stackelberg commitment-type (the commitment-type who plays the Stackelberg action in each period), and thereby establish a reputation of playing the Stackelberg action. In our subsequent analysis, we assume that the large player is sufficiently patient and use the two-stage precommitment equilibrium as the outcome under such infinitely-repeated games.

The result below compares the equilibrium profits of the large player in the precommitment equilibrium of G^1_∞ and G^2_∞.

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[9] More generally, in a repeated game, the commitment types play a pre-specified game strategy. Commitment types who play a fixed action are called simple commitment types, or action types.

THEOREM 2.1. ***(Seemingly-Beneficial Interventions that Induce Beneficial Actions)*** *Under Assumptions 2.1 – 2.4, if the large player's discount factor is sufficiently close to* 1 *and his actions are perfectly observable, then his payoff in the precommitment equilibrium of* G^2_∞ *is higher than his payoff in* G^1_∞*.*

Proof: Corollary 2.1 shows that if the large player's discount factor is sufficiently close to 1 and his actions in each period are perfectly observable, then his equilibrium NrPV converges to his Stackelberg payoff (Π^j_{ST}). Recall the definition of Π^j_{ST} from (2.1):

$$\Pi^j_{ST} = \max_{a_L \in \mathcal{A}_L} \left\{ \Pi^j(a_L, a_{S,A}) \text{ s.t. } a_{S,A} \in \mathcal{A}^j_{S,EQ}(a_L) \right\}$$

To prove Theorem 2.1, we are to show that $\Pi^1_{ST} \leq \Pi^2_{ST}$. We show the stronger result that for any $a_L \in \mathcal{A}_L$, $a'_{S,A} \in \mathcal{A}^1_{S,EQ}(a_L)$, $a''_{S,A} \in \mathcal{A}^2_{S,EQ}(a_L)$, the following holds:

$$\Pi^1(a_L, a'_{S,A}) \leq \Pi^2(a_L, a''_{S,A}).$$

From Assumption 2.1, we know that $\mathcal{A}^j_{S,EQ}(a_L)$, $j \in \{1,2\}$ exist. From Assumption 2.4, we have that $\mathcal{A}^1_{S,EQ}(a_L) \preccurlyeq \mathcal{A}^2_{S,EQ}(a_L)$. Therefore,

$$a'_{S,A} \preccurlyeq a''_{S,A}. \tag{2.2}$$

Using (2.2) and Assumption 2.3, we have that

$$\Pi^1(a_L, a'_{S,A}) \leq \Pi^1(a_L, a''_{S,A}). \tag{2.3}$$

Using Assumption 2.2, we have that

$$\Pi^1(a_L, a''_{S,A}) \leq \Pi^2(a_L, a''_{S,A}). \tag{2.4}$$

Combining (2.4) and (2.3), we have the required result. ■

REMARK 2.2. In this paper, we focus on seemingly-beneficial interventions that, if analyzed as one-shot games, hurt the large player. This is because of the OM applications that motivate our research – often, the surprising result in many OM papers involves demonstrating that seemingly-beneficial interventions hurt the large player. Consequently, in our key result (Theorem 2.1), we demonstrate the conditions under which seemingly-beneficial interventions benefit the large player under repeated interactions.

On the other extreme, there exist seemingly-beneficial interventions that *benefit* the large player in one-shot games but can *hurt* the large player over the long-run (under precommitment). First, we provide a set of sufficient conditions on the games G^1 and G^2 such that the seemingly-beneficial

		Small Player(s)	
		L	R
Large Player	U	2,3	0,2
	D	3,0	1, 1

Game G^1

		Small Player(s)	
		L	R
Large Player	U	5,5	1,6
	D	6,1	1.5,1.5

Game G^2

Figure 2 **Stage games G^1 and G^2. Game G^2 is seemingly-beneficial to game G^1, but does not induce beneficial actions. The precommitment equilibrium outcome under in the infinitely-repeated game G^2_∞ is inferior to that under G^1_∞.**

intervention, if analyzed as a one-shot game, benefits the large player; see Appendix C. Second, a seemingly-beneficial intervention that does not induce beneficial actions (i.e., if Assumption 2.4 does not hold) can lead to an inferior outcome for the large player under precommitment. We illustrate this using an example below.

EXAMPLE 2.2. The two games, G^1 and G^2, are identical to that in Example 2.1 except for the payoff matrices, which are shown in Figure 2. The Nash equilibrium in the one-shot games (both G^1 and G^2) involves the large player (resp., the small players) playing D (resp., R). We verify that Assumptions 2.1 – 2.3 hold:

(a) Consider the following action of the large player: $a_L = x \circ U + (1-x) \circ D$. We have that

$$\mathcal{A}^1_{S,EQ}(a_L) = \begin{cases} \{L\}, & \text{if } x > 1/2; \\ \{R\}, & \text{if } x < 1/2; \\ \mathcal{S}_S, & \text{if } x = 1/2, \end{cases}$$

while $\mathcal{A}^2_{S,EQ}(a_L) = \{R\}$.

(b) Game G^2 is seemingly beneficial to G^1, i.e., for any $a_L, a_{S,A}$, we have that $\Pi^1(a_L, a_{S,A}) \leq \Pi^2(a_L, a_{S,A})$.

(c) Consider two average plays $a'_{S,A} = x' \circ L + (1-x') \circ R$ and $a''_{S,A} = x'' \circ L + (1-x'') \circ R$ of the small players where $x', x'' \in [0,1]$ and $x' \leq x''$. We have that $a'_{S,A} \preccurlyeq a''_{S,A}$.

From (a) and (c) above, we have that $\mathcal{A}^2_{S,EQ}(a_L) \prec \mathcal{A}^1_{S,EQ}(a_L)$. Consequently, Assumption 2.4 does not hold. In the precommitment equilibrium, the outcome in G^1 involves the large player (resp., the small players) playing U (resp., L), while in G^2, the large player (resp., the small player) plays D (resp., R). ■

REMARK 2.3. In Appendix B, we extend our main model (homogenous small players) to accommodate heterogeneity among the small players. The key difference arises in Assumption 2.1, i.e., the definition of the symmetric best-response of the small players. We assume that the small players are associated with a type drawn from a distribution. Analogous to Assumption 2.1, we identify the symmetric-in-type best-response of the small players in Assumption B.1. Assumptions 2.2 – 2.4 are as stated in the main model. Under Assumption B.1 and Assumptions 2.2 – 2.4, our main result, namely Theorem 2.1, holds under heterogeneity among the small players, and the proof is identical.

3. Two Applications of Our Results

In this section, we consider two recent papers, Mu et al. (2014) and Gao and Su (2016), and demonstrate our main result from Section 2 in the context of their models.

3.1. Adulteration and Testing in Milk Supply Chains in Developing Economies

Mu et al. (2014, 2016) study the key economic forces that lead to milk adulteration in developing economies and provide recommendations to improve the quality of milk. The basic structure of the milk supply chain is as follows: A large intermediary (a *station*) procures raw milk from a population of small milk farmers (*producers*), mixes the milk, and sells it to a downstream firm (which, in turn, sells the processed milk to end consumers). The focus of their work is on the interaction between the station and the producers. The downstream firm tests the mixed milk supplied by the station and rewards it based on the quality of milk it supplies. On its part, the station procures milk from a population of producers and pays each producer based on the quality of the milk he supplies. Due to high testing costs, the station finds it too costly to individually test each producer; therefore, it randomly tests some (but not all) producers. This inability to test each producer creates an opportunity for individual producers to introduce adulteration – an individual producer who produces low-quality milk (that is cheaper to produce) but claims to produce high-quality milk may go untested and therefore enjoy a high payment.

In light of such practices, a key challenge faced by the milk stations is to design interventions, i.e., create incentives, that improve the quality of milk. Further, owing to their fundamental role in ensuring food safety and meeting developmental goals, governments in developing countries also have an incentive to improve the quality of milk. We consider one such intervention: The government supplements the producers with improved storage and refrigeration facilities, resulting in lowering the marginal cost of production for high-quality milk.[10] For simplicity, we hold the marginal cost

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[10] Mu et al. (2014) advocate for governmental interventions to support the various members of the milk supply chain. Quoting them "*...Over the years, the poor quality of milk has been a major concern in developing countries. Thus, one possibility is for the government to sponsor the mixed test. Government support could come either (a) by direct monetary assistance to the station, or (b) by a governmental office conducting the test at the station...*". Besides, Mu et al. (2016) propose governmental interventions in the context of multiple competing milk stations. Further, they recognize that the budget for governmental interventions in such countries is limited; therefore, they focus on interventions that do not burden the government. Quoting them, "*...have the government sponsor the bonus for the stations; the change is mild in that no bonus is actually paid in equilibrium. Therefore, this addition of governmental sponsorship of the bonus does not impose any burden on the government.*"

of production for low-quality milk fixed. Thus, as a result of the intervention, the profit margin on high-quality milk increases. It would be intuitive to conjecture that such an intervention would lead to an improvement in the quality of milk produced by an individual farmer, and thereby, an improvement in the quality of mixed milk. However, contrary to intuition, we show that the equilibrium quality of milk supplied by the producers does not change; our analysis is presented in Section 3.1.1 below. This is because, an individual producer anticipates that an improvement in the quality of milk by the producers encourages the station to lower its testing standards. Anticipating lower testing, the producers, in turn, have an incentive to lower the quality of milk they produce. In equilibrium, the quality of milk produced by the producers is independent of the difference in the marginal costs.

Notice that the conclusion above is based on the analysis of a single-shot simultaneous-move game, where the station chooses the testing level and the producers choose the quality of milk they produce. Motivated by the fact that producers and the station interact repeatedly over the long term, we examine the outcome under repeated interactions. Formally, we use the framework of a repeated game between a large player (the station) and a continuum of small players (producers), from Section 2. In this case, the station may have an incentive to establish a reputation that it tests more frequently than what the equilibrium in the single-shot model predicts; if the producers anticipate higher testing, they best-respond by improving the quality of milk produced. We find that repeated interactions allow the station to establish a reputation of testing with a high probability. Consequently, if the station is sufficiently patient, its short-term incentives to lower testing standards are dominated by its long-term incentives to establish a reputation of testing with high probability, thereby inducing the farmers to produce high-quality milk. An intervention that reduces the marginal cost of producing high-quality milk leads to an improvement in the quality of milk produced and therefore benefits the station.

We begin by formally describing the model.

3.1.1. A One-Shot Model of Testing: The basic setup of milk testing is as follows: A population of small, homogenous producers (mass normalized to 1) simultaneously and independently decide on the quality of milk they produce; an individual farmer decides on the quality of milk he produces. The quantity of milk that is produced by an individual producer is irrelevant to our analysis; all farmers are assumed to produce the same quantity and are paid based on the quality of milk they produce. For convenience, assume that the total quantity of milk procured by the station is one unit.

The unit economics for an individual producer and the station are as follows. Consider an individual producer, who chooses to produce milk of quality $\phi \in [0, \overline{\phi}]$. Let $w(\phi) = w_0 + w_1\phi$ denote the unit buying price at which the station procures milk of quality ϕ; we assume $w_1 > 0$. The payment to the producer is as follows: If the station tests the producer's milk, it pays the producer $w(\phi)$; otherwise, it pays the producer $w(\overline{\phi})$. Let $c(\phi) = c_0 + \frac{1}{2}c_2\phi^2$ denote the producer's unit production cost; we assume that $c_2 > 0$.

The station mixes the milk procured from the producers and hence the quality of mixed milk is the average quality chosen by the producers. The station sells the mixed milk to the downstream firm and is paid based on the quality of milk it supplies: Let $p(\phi) = p_0 + p_1\phi$ denote the selling price for milk of quality ϕ, $\phi \in [0, \overline{\phi}]$ (i.e., the price at which the station sells to the downstream firm). The quality of the mixed milk, denoted by ϕ_A, is the average quality of milk chosen by the producers. For an average quality ϕ_A, the selling price is $p_0 + p_1\phi_A$. The authors assume that $p_1 > w_1$; thus, the profit margin (to the station) increases in the quality of the milk. We assume that $\overline{\phi} = \frac{p_1}{c_2}$, i.e., $\overline{\phi}$ is the quality chosen by the integrated firm consisting of the station and the producers.

The station decides its testing strategy, i.e., whether to test individual producers. The station's strategy includes mixed actions: Let $x \in [0, 1]$ denote the station's testing strategy, i.e., the station tests an individual producer with probability x. Since the mass of producers is normalized to 1, the proportion (or mass) of producers tested is $1 \cdot x = x$. Testing is expensive for the station. Let t denote the unit cost of testing.[11] Then, corresponding to a strategy x (i.e., testing a proportion x of the producers), the station incurs a cost of tx.

Consider the problem that the station faces: Let $\hat{\phi}$ denote the station's belief about the (mean) quality of milk produced by an individual producer. Corresponding to an action, say x (probability of testing an individual producer) of the station, the (expected) profit of the station is

$$\Pi(x|\hat{\phi}) = \left[p_0 + p_1\hat{\phi}\right] - x\left[t + w_0 + w_1\hat{\phi}\right] - (1-x)\left[w_0 + w_1\overline{\phi}\right]. \tag{3.1}$$

The profit of the station is linear in x. Let $\phi^I = \overline{\phi} - \frac{t}{w_1}$. Thus,

$$x^*(\hat{\phi}_A) = \begin{cases} 0, & \text{if } \hat{\phi}_A > \phi^I; \\ 1, & \text{if } \hat{\phi}_A < \phi^I; \\ [0,1], & \text{if } \hat{\phi}_A = \phi^I. \end{cases} \tag{3.2}$$

(3.2) shows that it is optimal for the station to test all producers if the mean quality is below the threshold ϕ^I and test no producer if the mean quality exceeds this threshold. Intuitively, from the perspective of the station, the value of testing is greater if more producers produce low-quality milk. That is, if more producers produce low-quality milk, then testing helps the station in paying a high

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[11] Here, the unit cost refers to testing a unit mass of producers.

wage only to the producers of high-quality milk (instead of paying a high wage to *all* producers), and therefore testing is valuable. On the other hand, as more producers produce high-quality milk, the cost of testing outweighs the benefit that arises from identifying the low quality producers. Consequently, testing is less useful, and therefore the station does not test the producers.

Now, consider the problem faced by an individual producer, say i. Let $\hat{x}$ denote an individual producer's belief of the probability of testing. His profit from producing milk of quality ϕ is:

$$\pi(\phi|\hat{x}) = -\left[c_0 + \frac{1}{2}c_2\phi^2\right] + \hat{x}(w_0 + w_1\phi) + (1-\hat{x})(w_0 + w_1\overline{\phi}). \tag{3.3}$$

From (3.3), observe that for any fixed testing strategy of the station x, the profit of the producer, $\pi(\phi|\hat{x})$, is strictly concave in the quality ϕ. Therefore, corresponding to a fixed testing probability $\hat{x}$, the optimal quality of milk produced is:

$$\phi^*(\hat{x}) = \hat{x}\frac{w_1}{c_2}. \tag{3.4}$$

We now solve for the equilibrium actions of the station and the producers from the best-responses in (3.2) and (3.4). We focus on symmetric strategies for the producers. Let ϕ_i^{EQ} denote the equilibrium quality produced by an individual producer and x^{EQ} denote the equilibrium testing probability of the station.

Lemma 3.1. *The equilibrium strategies of the station and the producers are as follows:*

$$\begin{aligned} x^{EQ} &= \frac{\phi^I}{\Big(w_1/c_2\Big)} \quad \textit{and} \\ \phi_i^{EQ} &= \phi^I \ \forall i. \end{aligned}$$

Therefore, the equilibrium quality of the mixed milk is $\phi_A^{EQ} = \phi^I (= \overline{\phi} - \frac{t}{w_1})$.

Notice, from the above result, that the equilibrium quality of the mixed milk is *independent* of the costs of production. An important consequence of this result is the following: An intervention that results in a reduction in the cost of producing higher-quality milk, e.g., a governmental intervention that provides better storage and refrigeration facilities, does not lead to an improvement in the quality of the mixed milk. This is the conundrum that motivates our work in this section.

The intuition behind this result is as follows. Let the superscript 1 (resp., 2) denote the absence (resp., presence) of the intervention. An intervention, like the one above, leads to a decrease in $c(\phi)$: Suppose $c_0^2 \leq c_0^1$ and $c_2^2 \leq c_2^1$. Therefore, from (3.4), we have that ${\phi^*}^1(\hat{x}) \leq {\phi^*}^2(\hat{x})$. That is, for a fixed testing probability, the quality of milk produced by an individual producer is higher in the presence of the intervention (relative to that in its absence). However, by considering the strategic incentives of the station to test producers, we find that the station has an incentive to lower the testing probability in the presence of the intervention. This is because, the value from testing decreases if producers produce high-quality milk. Anticipating this decrease in the testing probability, an individual producer has an incentive to lower the quality of milk he produces. Therefore, in equilibrium, the quality of milk remains unchanged.

It is important to note that this outcome is a result of a one-shot model of interaction between a station and the producers. Motivated by the fact that the station interacts repeatedly with the producers over the long term, we study the outcome under repeated interactions and contrast it with the result above.

3.1.2. Testing under Repeated Interactions: Our Results

Consider the setting where the one-shot simultaneous move game described in Section 3.1.1 is played repeatedly. First, we verify that Assumptions 2.1 – 2.4 hold in the proposed intervention.

1. **(Symmetric Best-Response)** For any testing probability of the station, the symmetric best-response of the producers, given in (3.4), is unique.
2. **(Seemingly-Beneficial Intervention)** The intervention proposed above, where a social planner provides better storage and refrigeration equipment to lower the marginal cost of producing high quality milk, is (weakly) seemingly-beneficial. Consider a fixed testing probability x of the station and an average quality ϕ_A by the population of producers. The intervention does not affect the profit of the station, and hence the profit of the station remains fixed, i.e., Assumption 2.2 is satisfied with an equality.
3. **(Increasing Average Plays)** Consider a fixed testing probability, say x, chosen by the station. From (3.1), the profit of the station, $\Pi(x, \phi_A)$, is increasing in the average quality ϕ_A of the mixed milk. That is, for any testing strategy, the station prefers a higher quality of the mixed milk.
4. **(Interventions Induce Higher Average Plays)** Consider a fixed testing probability x of the station. From (3.4), the best-response of the producers (i.e., the quality of milk) is higher in the presence of the intervention, i.e., $\phi^{*1}(x) \leq \phi^{*2}(x)$.

Therefore, we use the results from Section 2 to analyze this repeated game between the station and the producers. In any period, the payoff-relevant variables for the producers (resp., station) are their beliefs on the testing probability (proportion of producers who produce high quality milk) – the producers best-respond to the anticipated testing strategy of the station.

In the two-stage precommitment equilibrium, the station can be assumed to commit to a single testing level $x \in [0, 1]$. The producers effectively play their best response to the station's committed strategy. Therefore, their best response, $\phi^*(x)$, is given in (3.4). The "Stackelberg" profit (NrPV) of the station is:

$$\Pi(x|\phi_A = \phi^*(x)) = \left(p_0 + p_1 x \frac{w_1}{c_2}\right) - x\left(t + w_0 + w_1 x \frac{w_1}{c_2}\right) - (1-x)(w_0 + w_1\overline{\phi}). \tag{3.5}$$

Therefore, the equilibrium testing probability of the station is

$$x^* = x^{EQ} + \frac{t/(2w_1)}{w_1/c_2}. \tag{3.6}$$

Further, $\phi^* = \phi^{EQ} + \frac{t}{2w_1}$. Observe that $x^* > x^{EQ}$ and $\phi^* > \phi^{EQ}$. This is because, if the station could establish a reputation of testing with a high probability, then the producers respond by producing high quality milk. In the one-shot model, the station has a short-term incentive to lower its testing standards in the presence of the intervention. However, under repeated interactions, if the station is sufficiently patient, then these short-term incentives are dominated by an incentive to establish a reputation of testing with a high probability. Therefore, in equilibrium, the station establishes a reputation that its testing standard strictly exceeds x^{EQ}. Further, the one-shot model's prediction of the station's equilibrium profit is independent of c_0 and c_2. However, under repeated interactions, the equilibrium profit of the station increases as c_2 decreases, i.e., the intervention indeed benefits the station.

Remark 3.1. In Appendix A, we illustrate how the station is able to establish a reputation of testing with a high probability in an infinitely-repeated game between the station and the producers. We assume that producers adaptively learn the testing strategy of the station over time using an exponential learning rule. This approach has been commonly employed in the OM literature – in particular, in papers that explicitly model strategic consumer behavior in repeated games; see, e.g., Su and Zhang (2009), Liu and Van Ryzin (2011).

Remark 3.2. Recall, from the general model in Section 2.5, that for the large player to be able to establish a reputation that he is a Stackelberg type, we require that his actions are perfectly observed. In the context of the model above, we require that the testing strategy of the station is observed perfectly, and the actions of an individual farmer are not part of the observable history. These are both reasonable in the context of our model: The lack of sophisticated technology at the station to monitor the past actions of the farmers implies that their actions are not part of the history. Besides, we assume that social interactions among the farmers allows for the station's actions to be perfectly observed.

3.2. Information Provision Mechanisms in Omnichannel Retail

Our discussion in this section is motivated by Gao and Su (2016), who study how retailers operating in an omni-channel environment use information-provision mechanisms to help consumers resolve their uncertainty about a product's valuation before purchase. In particular, they study two mechanisms that have gained prominence in recent years: (a) physical showrooms and (b) virtual showrooms, and compare the performance of a "traditional" retailer (who does not adopt any mechanism to provide information to consumers *a priori*) to a retailer who uses either of these mechanisms.

Conventional wisdom suggests that the use of such information-provision mechanisms will not only help consumers in resolving their valuation uncertainty before purchase, but will also benefit the retailer by lowering the costs incurred due to consumer returns. Thus, one would intuitively expect such an intervention from the retailer to lead to a "win-win" outcome, i.e., improve both the retailer's profit and the consumers' utility. Surprisingly, in an important result of their paper, Gao and Su (2016) show that such mechanisms can potentially *hurt* the retailer and the consumers, considering the strategic incentives of both the parties under the mechanisms. This is the conundrum that motivates this section of our paper. Briefly, the intuition behind the results in Gao and Su (2016) is as follows: Relative to their absence, physical showrooms reduce in-store inventory and hence increase consumers' stock-out risk, thereby discouraging store patronage and diverting consumers to go online. This leads to more returns and hurts the retailer when the cost associated with returns is high. Virtual showrooms reduce product-value uncertainty and make online shopping more appealing. This leads to more returns, and hurts the retailer when the online profit margin is low.

It is important to note that the analysis in Gao and Su (2016) is based on a single-shot, simultaneous-move game, in which the retailer decides the in-store stocking level, and the consumers decide whether to shop in-store or online. Motivated by the fact that retailers and their consumers typically interact with each other repeatedly over the long term, we examine the conundrum under repeated interactions.[12] We now summarize the models and the results of Gao and Su (2016) that are relevant to our analysis.

3.2.1. A One-Shot Model of Information Provision in Omni-Channel Retail

The authors first consider a "traditional" retailer who provides no information to consumers before purchase (the base model). Subsequently, they consider a retailer operating with a physical showroom in-store, or a virtual showroom online.

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[12] By *repeated interations*, we mean that the single-period game is played multiple times across time periods, for a different product – e.g., a different generation of a product – in each period.

Base Model A retailer (he) operating in an omni-channel environment sells a new product using two channels: in-store (offline) and online. The store channel is modeled as follows: The price of a unit item is p, and the cost for stocking an item is c. Before the demand is realized, the retailer decides the in-store stocking level q. Leftover units have no value. The online channel is modeled exogenously as follows: The retailer obtains a net profit margin of w for each unit sold online successfully (i.e., not returned), and incurs a net loss of r if it is returned (i.e., an unsuccessful transaction).[13]

The market size, denoted by D, is random and follows a distribution $F(\cdot)$. Consumers are ex-ante homogenous and uncertain about their valuation: a fraction θ have a positive value v for the product ("high-type" consumers), and the remaining $(1-\theta)$ have value 0 ("low-type" consumers). They realize their valuation after examining the product in-store; otherwise, they learn their valuation after purchase. Each consumer makes a choice between shopping online or visiting the store by comparing her payoff from both the channels. The consumer-utility model is as follows: If she buys from the online channel, she incurs a hassle cost h_o, and realizes her valuation only after receiving the product. If she likes the product (high-type), she keeps the product, and receives a payoff $v-p-h_o$; if she dislikes the product (low-type), she returns the product. Returns are costly to both the retailer and the consumer: Each returned unit generates a loss $r>0$ for the firm and a return-hassle cost $h_r>0$ to the consumer.[14] A consumer's expected payoff from buying online, denoted by u_o, is then given by:

$$u_o = -h_o + \theta[v-p] + (1-\theta)[-h_r].$$

If the consumer goes to the store first, she incurs a store-hassle cost h_s, and subsequently, one of the following outcomes occur: If the store is in-stock, then she evaluates the product and realizes her valuation (either 0 or v) immediately. If she is a high-type consumer, then she purchases the product and receives a payoff $v-p$. If she is a low-type consumer, then she does not purchase the product and receives a payoff 0. If the store is out-of-stock, then she goes online, and receives the online payoff u_o. All the parameters are fixed and common knowledge. Conditional on consumers adopting a symmetric mixed strategy ϕ (the probability of visiting the store), the in-stock probability corresponding to a stocking-level q is given by

$$\xi(\phi, q) = \frac{\mathbb{E}\left[\min\left\{\frac{q}{\theta}, \phi D\right\}\right]}{\mathbb{E}[\phi D]}. \tag{3.7}$$

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[13] The assumption here is that any order placed online will be satisfied, and therefore, the authors abstract away from modeling stocking decisions for the online channel.

[14] The assumption here is that $h_r < p$, so that low-type consumers strictly prefer returning the product.

Therefore, a consumer's expected payoff from visiting the store, denoted by u_s, is given by

$$u_s = -h_s + \xi\Big[\theta(v-p)\Big] + (1-\xi)\big[u_o\big].$$

The profit of the retailer corresponding to an in-store stocking level of q, when consumers adopt a symmetric mixed strategy $\phi \in [0,1]$, is

$$\begin{aligned}\pi_B(q|\phi) &= p\theta\,\mathbb{E}\left[\min\left\{\phi D, \frac{q}{\theta}\right\}\right] - cq + \big(w\theta - r(1-\theta)\big)\,\mathbb{E}\left[\left(\phi D - \min\left\{\phi D, \frac{q}{\theta}\right\}\right) + (1-\phi)D\right] \\ &= \left(p - w + \frac{r(1-\theta)}{\theta}\right)\mathbb{E}\big[\min\{\theta\phi D, q\}\big] - cq + \pi_o, \end{aligned} \tag{3.8}$$

where $\pi_o = (w\theta - r(1-\theta))\,\mathbb{E}[D]$. The equilibrium of this game is presented in Proposition 1 of Gao and Su (2016).

Physical Showrooms The retailer with an in-store physical showroom is modeled as follows: Consumers can always inspect the product in a store with a physical showroom, despite a stock-out (due to the availability of a display product). The parameters of the retailer and the consumers are identical to those in the base model. A consumer's payoff from choosing the online channel is identical to that in the base model. Her payoff from choosing the store channel is

$$u_s = -h_s + \xi\Big[\theta(v-p)\Big] + (1-\xi)\Big[\theta(-h_o + v - p)\Big].$$

The difference from her corresponding payoff in the base model arises in the second term of the above expression, when a consumer goes to the store and experiences a stock-out: She realizes her valuation by inspecting the display product and, hence, only a high-type consumer goes online (in the base model, all consumers who face a stock-out in-store go online). In essence, physical showrooms do not affect the payoff of high-type consumers but help low-type consumers realize their valuation even during a stock-out.

The profit of the retailer corresponding to an in-store stocking level of q, when consumers adopt a symmetric mixed strategy $\phi \in [0,1]$, is

$$\begin{aligned}\pi_{PS}(q|\phi) &= p\theta\,\mathbb{E}\left[\min\left\{\phi D, \frac{q}{\theta}\right\}\right] - cq + w\theta\,\mathbb{E}\left[\phi D - \min\left\{\phi D, \frac{q}{\theta}\right\}\right] + \big(w\theta - r(1-\theta)\big)\,\mathbb{E}\big[(1-\phi)D\big] \\ &= \big(p-w\big)\,\mathbb{E}\big[\min\{\theta\phi D, q\}\big] - cq + \pi_o + r(1-\theta)\,\mathbb{E}[\phi D]. \end{aligned} \tag{3.9}$$

The equilibrium of this game is presented in Proposition 2 of Gao and Su (2016). Further, their Proposition 3 compares the performance of a retailer in the base model and one with a physical showroom, and shows that *physical showrooms hurt the retailer when the value of θ is moderate.* The intuition is as follows: Conditional on consumers visiting the store, the retailer operating an in-store physical showroom finds it optimal to lower the in-store stocking level (vis-á-vis the base model). Anticipating the resultant increase in the stockout risk, a higher proportion of consumers choose to shop online. This increases consumer returns and hurts the retailer when the cost associated with returns is high.

Virtual Showrooms The retailer with a virtual showroom in the online channel is modeled as follows: All consumers first inspect the product virtually and receive an *imperfect* signal of their valuations. All high-type consumers remain interested in the product, while only a fraction $(1-\alpha)$ of the low-type consumers remain interested (i.e., a fraction $\alpha \in [0,1]$ of the low-type consumers realize their valuation through the signal).[15] Therefore, the total demand size, denoted by D', is $(1-\alpha(1-\theta))D$, and the posterior probability of a high-type consumer, denoted by $\theta'(>\theta)$ and derived from Bayesian updating, is $\frac{\theta}{1-\alpha(1-\theta)}$. All other parameters of the retailer and the consumers are identical to those in the base model.

The profit of the retailer corresponding to an in-store stocking level of q, when consumers adopt a symmetric mixed strategy $\phi \in [0,1]$, is

$$\begin{aligned}\pi_{VS}(q|\phi) &= p\theta' \mathbb{E}\left[\min\left\{\phi D', \frac{q}{\theta'}\right\}\right] - cq + \big(w\theta' - r(1-\theta')\big)\mathbb{E}\left[\left(\phi D' - \min\left\{\phi D', \frac{q}{\theta'}\right\}\right) + (1-\phi)D'\right] \\ &= \left(p - w + \frac{r(1-\alpha)(1-\theta)}{\theta}\right)\mathbb{E}\left[\min\{\theta\phi D, q\}\right] - cq + \pi_o + r\alpha(1-\theta)\,\mathbb{E}[D]. \qquad (3.10)\end{aligned}$$

The equilibrium of this game is presented in Proposition 4 of Gao and Su (2016). In their Proposition 5, the authors compare the performance of a retailer in the base model and one with a virtual showroom, and show that *virtual showrooms hurt the retailer when the value of θ is moderate and w is low*. The intuition is as follows: A virtual showroom makes online purchasing more appealing and increases total returns, which hurts the retailer when the online profit margin is low.

We now analyze repeated interactions between the retailer and consumers, over the long term.

3.2.2. Repeated-Interaction Model of Omni-Channel Retail: Our Results

Consider the setting where the one-shot simultaneous-move game(s) described in Section 3.2.1 is repeated over an infinite number of periods. Specifically, in each period, the retailer's decision is the in-store stocking level for the product under consideration in that period, while each consumer decides to shop in-store or online (possibly mixed). In any period, the public histories of the game include the retailer's stocking level and the proportion of consumers who shop in-store in each period, until that period. The players maximize their individual NrPVs. Our goal in this section is to compare the retailer's equilibrium NrPV in the three cases, viz., the base model, the physical showroom, and the virtual showroom. Recall from our discussion of the precommitment equilibrium (Section 2.5) that the retailer's equilibrium NrPV converges to his Stackelberg payoff as his discount factor approaches 1. Accordingly, we compare the Stackelberg payoff of the retailer in the three cases.

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[15] Gao and Su (2016) interpret α as the degree of informativeness of the signal.

The main results of this section are Lemmas 3.2 and 3.3. In contrast to Gao and Su (2016), Lemma 3.2 shows that, *under repeated interactions, a physical showroom always benefits the retailer.* Consistent with Gao and Su (2016), Lemma 3.3 shows that *under repeated interactions, a virtual showroom can hurt the retailer.* We then explore the fundamental difference between these two mechanisms that leads to these contrasting conclusions.

Base Model We first define the following quantities that help in our analysis. Let ξ_B^I denote the following in-stock probability:

$$\xi_B^I = \min\left\{1, \frac{h_s}{h_o + (1-\theta)h_r}\right\}.$$

Intuitively, ξ_B^I is the in-stock probability that makes a consumer indifferent between choosing the online and the store channel in the base model. Using (3.7), let q_B^I be implicitly defined as $\xi(1, q_B^I) = \xi_B^I$. When all consumers visit the store, q_B^I is the in-store stocking level that leads to an in-stock probability of ξ_B^I.

In the two-stage precommitment equilibrium of the base model, the retailer can be assumed to commit to a single stocking level $q \geq 0$. Since all consumers then choose to shop in-store or online simultaneously, the consumers effectively play a best response to the retailer's committed strategy, and hence each consumer chooses to go to the store with a probability $\phi_B^*(q) = \min\{1, \frac{q}{q_B^I}\}$. The intuition is as follows: Recall that q_B^I is the stocking-level in store that makes a consumer indifferent between the two channels when all other consumers visit the store channel. Therefore, for any stocking level q larger than q_B^I, consumers strictly prefer shopping in-store. If the firm stocks $q = 0$, then it is optimal for any consumer to shop online. For intermediate values of q, consumers use a mixed strategy. Since each consumer chooses to shop in-store with a probability ϕ_B^*, the aggregate proportion of consumers who shop in-store is ϕ_B^*.

The Stackelberg profit of the retailer (i.e., the payoff under his best commitment strategy), using (3.8), is then given by $\max_{q \geq 0} \pi_B(q|\phi_B^*(q))$.

Physical Showroom Similar to the two-stage precommitment equilibrium in the base model above, we can obtain the precommitment equilibrium when the retailer operates an in-store physical showroom. In this case, we begin with the following definitions. Let

$$\xi_{PS}^I = \max\left\{0, \min\left\{1, \frac{h_s - (1-\theta)(h_o + h_r)}{\theta h_o}\right\}\right\}.$$

Define q_{PS}^I implicitly as $\xi(1, q_{PS}^I) = \xi_{PS}^I$. The intuition for ξ_{PS}^I (resp., q_{PS}^I) is identical to ξ_B^I (resp., q_B^I) in the base model, except that these quantities are defined for the physical showroom model.

Observe that $\xi^I_{PS} \leq \xi^I_B$ and $q^I_{PS} \leq q^I_B$: This is because, ceteris paribus, physical showrooms remove product-valuation uncertainty, and hence a lower stocking level is sufficient to incentivize the consumers to visit the store vis-á-vis the base model.

Suppose the retailer commits to an in-store stocking level $q \geq 0$. Responding best to this stocking level, consumers choose to shop in-store with a probability $\phi^*_{PS}(q) = \min\{1, \frac{q}{q^I_{PS}}\}$. The Stackelberg payoff of the retailer, from (3.9), is then $\max_{q\geq 0} \pi_{PS}(q|\phi^*_{PS}(q))$. We now compare the profit of the retailer in the precommitment equilibrium of the base model with that in the presence of a physical showroom. We first verify that assumptions 2.1 – 2.4 hold under physical showrooms:

1. **(Symmetric Best-Response)** The best-response of the consumers corresponding to a stocking level, say q, in the base model (resp., in the presence of a physical showroom) is given by $\phi^*_B(q) = \min\{1, \frac{q}{q^I_B}\}$ (resp., $\phi^*_{PS}(q) = \min\{1, \frac{q}{q^I_{PS}}\}$) and is unique.
2. **(Seemingly-Beneficial Intervention)** Physical showrooms are seemingly-beneficial interventions, relative to the base model. That is, for a fixed proportion of consumers choosing to shop in-store, and a fixed in-store stocking quantity, it is straightforward to see, by comparing (3.8) and (3.9), that the profit of the retailer is higher with a physical showroom, i.e., for a fixed $\phi \in [0,1]$ and $q \geq 0$, we have

$$\pi_B(q|\phi) \leq \pi_{PS}(q|\phi).$$

   Therefore, for any $\phi \in [0,1]$, we have

$$\max_{q\geq 0} \ \pi_B(q|\phi) \quad \leq \quad \max_{q\geq 0} \ \pi_{PS}(q|\phi).$$

3. **(Increasing Average Plays)** Next, observe that both $\pi_B(q|\phi)$ and $\pi_{PS}(q|\phi)$ are (weakly) increasing in ϕ for a given value of q – that is, for a given in-store stocking level q, the retailer benefits with a higher proportion of consumers shopping in-store, in both the cases. Further, Remark 2.1 is applicable to this setting.
4. **(Interventions Induce Higher Average Plays)** Finally, from the discussion at the start of this section, we know that $q^I_B \geq q^I_{PS}$. Hence, for any fixed $q \geq 0$, we have

$$\phi^*_B(q) \quad \leq \quad \phi^*_{PS}(q).$$

   In words, for a fixed stocking level of the retailer, a higher proportion of consumers choose to shop in-store when the retailer operates a physical showroom.

Combining these arguments, we have

$$\max_{q\geq 0}\ \pi_B(q|\phi_B^*(q)) \quad \leq \quad \max_{q\geq 0}\ \pi_{PS}(q|\phi_{PS}^*(q)),$$

i.e., the retailer's profit in the precommitment equilibrium is always higher with the physical showroom. Thus, we have the following result.

LEMMA 3.2. *The profit of the retailer operating a physical showroom in the precommitment equilibrium is higher than that in the base model.*

The intuition behind this result is as follows: While in a one-shot model, consumers might be skeptical of going to the store anticipating a low stocking level, in a repeated setting, the retailer has an incentive to persistently have a high in-store stocking-level (i.e., his Stackelberg strategy), in order to establish a reputation for high service levels. While the retailer takes time and incurs a cost to establish such a reputation, this cost becomes negligible if he is sufficiently patient. When he establishes such a reputation, a physical showroom – which incentivizes consumers to shop in-store – benefits the retailer.

Virtual Showrooms Similar to the analysis above, we now obtain the precommitment equilibrium for the case where the retailer operates a virtual showroom in the online channel. Let

$$\xi_{VS}^I = \min\left\{1, \frac{h_s}{h_o + (1-\theta')h_r}\right\},$$

and define q_{VS}^I implicitly by $\xi(1, q_{VS}^I) = \xi_{VS}^I$. As before, the intuition for ξ_{VS}^I and q_{VS}^I is similar. Observe that $\xi_{VS}^I \geq \xi_B^I$, and $q_{VS}^I \geq q_B^I$: This is because all consumers receive an informative signal which reduces their product value uncertainty, and hence they have a greater incentive to shop online vis-á-vis the base model.

Suppose the retailer commits to an in-store stocking level $q \geq 0$. The best response of the consumers to a stocking level q is then $\phi_{VS}^*(q) = \min\left\{1, \frac{q}{q_{VS}^I}\right\}$. The Stackelberg payoff of the retailer, using (3.10) is then given by $\max_{q\geq 0} \pi_{VS}(q|\phi_{VS}^*(q))$.

We now compare the profit of the retailer in the precommitment equilibrium of the base model with that in the virtual showroom. Interestingly, we find that in the precommitment equilibrium, the profit of the retailer with a virtual showroom can be lower than his profit in the base case. This finding is consistent with that in Gao and Su (2016); we illustrate it below using a simple numerical example.

Example 3.1. Consider the virtual showroom mechanism and let $\alpha = 1$, i.e., the virtual showroom completely resolves the product-valuation uncertainty of the consumers. Suppose $h_o < h_s$: Since all the consumers that remain in the market are of high type, and the online hassle cost is lower than the store hassle cost, they shop online. Therefore, in the precommitment equilibrium, the retailer stocks 0 units in-store. Now, suppose that $D \sim \exp(\delta)$, $p = \$30$, $w = \$15$, $r = \$1$, $\theta = 0.2$, $c = (p - w + r\frac{1-\theta}{\theta})\frac{1}{e} \approx \$6.99$, $\frac{h_s}{h_o + (1-\theta)h_r} = 1 - \frac{1}{e}$. Then, $\pi_{VS} = w\theta\,\mathbb{E}[D] = \frac{3}{\delta}$. Consider the base case. Here, in the precommitment equilibrium, the retailer stocks $\frac{\theta}{\delta}$. Hence, $\pi_B = \frac{\theta p - w\theta + r(1-\theta)}{\delta}(1 - \frac{2}{e}) + \frac{w\theta - r(1-\theta)}{\delta} = \frac{3.204}{\delta}$ $(> \pi_{VS})$. ■

We state the result below.

Lemma 3.3. *The profit of the retailer operating a virtual showroom in the precommitment equilibrium can be lower than that in the base model.*

Lemmas 3.2 and 3.3 establish contrasting conclusions: Under repeated interactions, physical showrooms always benefit the retailer while virtual showrooms can hurt the retailer. We now examine a key difference between these two information-provision mechanisms that leads to these disparate outcomes.

Key Difference between Physical and Virtual Showrooms: Regardless of whether the retailer uses a physical showroom in-store or a virtual showroom online, Assumptions 2.1 – 2.3 hold. However, Assumption 2.4 holds *only with the physical showroom*: That is, for any fixed stocking level $q \geq 0$, in-store, relative to the base case, the proportion of consumers shopping in-store is higher with the physical showroom, i.e., $\phi^*_B(q) \leq \phi^*_{PS}(q)$. In contrast, with virtual showrooms, we have $\phi^*_{VS}(q) \leq \phi^*_B(q)$.

Thus, the key difference between the two mechanisms can be succinctly summarized as follows: For a fixed stocking level in-store, the retailer prefers to have a higher proportion of consumers shop in-store (from (3.9) and (3.10)). However, a virtual showroom makes online shopping more appealing and, hence, more consumers shop online (relative to the base case). In contrast, with a physical showroom, when the retailer commits to a stocking level in-store, more consumers shop in-store (relative to the base case). This fundamental difference leads to contrasting outcomes for the retailer.

We conclude by highlighting our main message.

4. Substantive and Managerial Implications

From a substantive perspective, our analysis identifies two characteristics that are fundamental in determining whether an intervention will always help or can possibly hurt the firm:

- *Nature of the Intervention*: Fixing the firm's action, does the introduction of the intervention induce consumers to take actions that provide higher benefit to the firm? If affirmative, the intervention can be said to induce beneficial actions. For instance, in the context of milk adulteration discussed in Section 3.1, for a fixed testing strategy of the milk station, an intervention that reduces the marginal cost of producing high-quality milk (e.g., a governmental intervention that provides better storage and refrigeration equipment to the producers) induces them to produce higher quality milk.
- *Extent of Interaction*: Do the consumers interact with the firm repeatedly and, if yes, is the firm sufficiently patient? For instance, in Section 3.1, it is reasonable to assume that the producers (small farmers) interact with the milk station repeatedly over the long-term.

From a managerial perspective, when a one-shot interaction is appropriate, then *irrespective of the nature of the intervention*, a seemingly-beneficial intervention can hurt the firm. On the other hand, for environments in which repeated interactions are natural, regardless of the outcome under a one-shot interaction, interventions that induce beneficial actions from the consumers *always benefit* the firm. However, if a seemingly-beneficial intervention does not induce beneficial actions from the consumers, then the intervention can hurt the firm under repeated interactions even though it benefits the firm under a one-shot interaction.

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Appendix A: An Illustration of the Equilibrium Outcome under Repeated Interactions with Adaptive Learning

Consider the case where the action spaces are compact subsets of $\mathbb{R}$ (i.e., $\mathcal{A}_L, \mathcal{A}_S \subseteq \mathbb{R}$), and the payoffs of the players, $\Pi^j(a_L, a_{S,A})$ and $\pi^j(a_L, a_{S,A}, a_S)$, are continuous and differentiable in their arguments. Let $\hat{a}_L$ denote the "anticipated" action of the large player by the small players. The small players play the symmetric best-response, $a^j_{S,EQ}(\hat{a}_L)$. The payoff of the large player in game G^j, $j \in \{1,2\}$ is $\Pi^j(a_L, a_{S,EQ}(\hat{a}_L))$. With a mild abuse of notation, we denote $\Pi^j(a_L, a_{S,EQ}(\hat{a}_L))$ by $\Pi^j(a_L, \hat{a}_L)$.

Consider the repeated game G^j_∞, where the large player interacts repeatedly with the small players over periods $t = 1, 2, \ldots$. In any period t, the small players anticipate an action $(\hat{a}_L)_t$, and play the symmetric best-response $a^j_{S,EQ}((\hat{a}_L)_t)$. The large player chooses an action $(a_L)_t$. At the end of the period, the small players perfectly observe the action of the large player $(a_L)_t$ and update their belief for the next period. In period $t+1$, the anticipated action of the large player by the small players is $(\hat{a}_L)_{t+1} = \alpha(a_L)_t + \overline{\alpha}(\hat{a}_L)_t$, where $0 \leq \alpha \leq 1$ and $\overline{\alpha} = 1 - \alpha$.

The large player playing this repeated game faces the following dynamic optimization problem, with the objective of maximizing his NrPV over an infinite horizon.

$$\begin{aligned} &\max_{(a_L)_t \in \mathcal{A}_L} \quad (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1}\Pi((a_L)_t, (\hat{a}_L)_t) \\ \text{s.t.} \quad &(\hat{a}_L)_{t+1} = \alpha(a_L)_t + \overline{\alpha}(\hat{a}_L)_t. \end{aligned}$$

We write the Bellman's equation for the large player's optimization problem:

$$V^j(\hat{a}_L) = \max_{a_L \in \mathcal{A}_L} \Big[\Pi^j(a_L, \hat{a}_L) + \delta V^j(\alpha a_L + \overline{\alpha}\hat{a}_L)\Big]. \tag{A.1}$$

Let ${a^*_L}^j(\hat{a}_L)$ denote the maximizer to (A.1). Then, for any $\hat{a}_L$, the following two conditions hold:

1. Since ${a^*_L}^j(\hat{a}_L)$is the maximizer to (A.1), we use first-order conditions to obtain the following:

$$\left[\frac{\partial \Pi^j(a_L, \hat{a}_L)}{\partial a_L} + \delta\alpha {V^j}'(\alpha a_L + \overline{\alpha}\hat{a}_L)\right]\Bigg|_{a_L = a^*_L(\hat{a}_L)} = 0 \tag{A.2}$$

2. From the envelope theorem, we have:

$${V^j}'(\hat{a}_L) = \frac{\partial \Pi^j(a_L, \hat{a}_L)}{\partial \hat{a}_L} + \delta\overline{\alpha}{V^j}'(\alpha a_L + \overline{\alpha}\hat{a}_L). \tag{A.3}$$

Suppose there exists a fixed point $\dot{a}^j_L$ to ${a^*_L}^j(\cdot)$, i.e., ${a^*_L}^j(\dot{a}^j_L) = \dot{a}^j_L$. Then, for any initial belief $(\hat{a}_L)_1$, we have that $(\hat{a}_L)_t \rightarrow \dot{a}_L$. Substituting $\hat{a}_L = \dot{a}_L$ in (A.2) and (A.3), we have:

$$\left[\delta\alpha\frac{\partial \Pi^j(a_L, \hat{a}_L)}{\partial \hat{a}_L} + (1-\delta\overline{\alpha})\frac{\partial \Pi^j(a_L, \hat{a}_L)}{\partial a_L}\right]\Bigg|_{\substack{a_L = {a^*_L}^j(\hat{a}_L), \\ \hat{a}_L = \dot{a}_L}} = 0. \tag{A.4}$$

Observe that, on the one end, if $\delta = 0$, then the solution to $\dot{a}_L$ in (A.4) corresponds to the Nash equilibrium of the one-shot game, while on the other end, if $\delta = 1$, we obtain the optimal commitment of the large player. An intermediate value of δ leads to an intermediate value of $\dot{a}_L$.

Now, consider the example discussed in Section 3.1. In this case, (A.4) becomes:

$$\Big[\alpha\delta\tfrac{w_1}{c_2}\big(p_1 - w_1\dot{x}\big) + (1-\overline{\alpha}\delta)\big(-t + w_1(\overline{\phi} - t\tfrac{w_1}{c_2})\big)\Big] = 0$$
$$\Rightarrow \dot{x}(\delta) = \frac{\dot{\phi}(\delta)}{\left(\frac{w_1}{c_2}\right)}, \text{ where } \dot{\phi}(\delta) = \tfrac{\alpha\delta}{\alpha\delta+(1-\overline{\alpha}\delta)}\overline{\phi} + \tfrac{1-\overline{\alpha}\delta}{\alpha\delta+(1-\overline{\alpha}\delta)}\big(\overline{\phi} - \tfrac{t}{w_1}\big). \tag{A.5}$$

Notice that $\dot{\phi}(\delta)$ is increasing in δ, and therefore $\dot{x}(\delta)$ is also increasing in δ. That is, as the station is increasingly patient (i.e., values future payoffs), its testing probability increases. Further, as $\delta \to 1$, we have that $\dot{x}(\delta) \to x^*$ as shown in (3.6).

Appendix B: Heterogeneity of Small Players

Consider an identical framework as explained in Section 2.1, except for the following difference to allow for heterogeneity among the small players. The key difference arises in Assumption 2.1. Suppose that each small player (in the continuum of small players) draws a type $\theta \in \Theta$ in period 0 according to $p(\cdot)$. Θ is either a finite set, or a compact subset of $\mathbb{R}$. If Θ is a finite set, then $p(\cdot)$ denotes the p.m.f of θ; otherwise $p(\cdot)$ denotes the p.d.f. of θ. The (pure) action space of all small players is identical and is denoted by $\mathcal{A}_S$; $\mathcal{S}_S = \Delta(\mathcal{A}_S)$. The actions of all the small players induces a population action distribution over $\mathcal{A}_S$, i.e., $a_{S,A} \in \mathcal{S}_S$. The payoff of a small player of type θ in game G^j is denoted by $\pi^j_\theta(a_S, a_L, a_{S,A})$, while that of the large player is given by $\Pi^j(a_L, a_{S,A})$. In particular, as stated in Section 2.3, we assume that the large player's payoff depends on his action and the average of the small players' actions, while the payoff of each small player depends on his type, his action, the large player's action and the average of the small players' actions.

Consider the following symmetric (in-type) action profile: $\boldsymbol{a}_S = (a_{S,\theta})_{\theta\in\Theta}$, $a_{S,\theta} \in \mathcal{S}_S$. That is, all small players of type θ play action $a_{S,\theta}$. The average play of the small players corresponding to $\boldsymbol{a}_S$ is given by

$$\breve{a}_{S,A}(\boldsymbol{a}_S) = \begin{cases} \sum_\theta p(\theta) a_{S,\theta}, & \text{if } \Theta \text{ is finite;} \\ \int_{\theta\in\Theta} p(\theta) a_{S,\theta} d\theta, & \text{if } \Theta \text{ is a compact subset of } \mathbb{R}. \end{cases}$$

Therefore, $\breve{a}_{S,A} \in \mathcal{S}_S$. Consider game G^j: The set of best-responses of an individual small player of type θ, corresponding to a large player's action $a_L \in \mathcal{S}_L$ and average play of small players $a_{S,A} \in \mathcal{S}_S$ is denoted by $\mathcal{A}^j_{S,\theta,B}(a_L, a_{S,A})$.

$$\mathcal{A}^j_{S,\theta,B}(a_L, a_{S,A}) = \arg\max_{a_S \in \mathcal{S}_S} \pi^j_\theta(a_S, a_L, a_{S,A}).$$

Assumption B.1. ***(Symmetric-In-Type Best Response)*** *For any action $a_L \in \mathcal{S}_L$ of the large player, define the set of symmetric-in-type best response $\hat{\mathcal{A}}^j_{S,EQ}(a_L) \subseteq \mathcal{S}_S$ as follows:*

$$\hat{\mathcal{A}}^j_{S,EQ}(a_L) = \big\{a_{S,A} \in \mathcal{S}_S : a_{S,A} = \breve{a}_{S,A}(\boldsymbol{a}_S) \text{ where } \boldsymbol{a}_S = (a_{S,\theta})_{\theta\in\Theta} \text{ and } a_{S,\theta} \in \mathcal{A}^j_{S,\theta,B}(a_L, a_{S,A}) \text{ for each } \theta \in \Theta\big\}.$$

We assume that $\hat{\mathcal{A}}^j_{S,EQ}(a_L)$ is non-empty.

In general, $\hat{\mathcal{A}}^j_{S,EQ}(a_L)$ may not be singleton. We define:

$$\mathcal{A}^j_{S,EQ}(a_L) = \arg\min_{a_{S,A} \in \hat{\mathcal{A}}^j_{S,EQ}(a_L)} \Pi^j(a_L, a_{S,A}).$$

Assumptions 2.2 – 2.4 are as given in Section 2.3. Under Assumption B.1 and Assumptions 2.2 – 2.4, our main result, Theorem 2.1, holds for this extension and the proof is identical.

Appendix C: A Sufficient Condition for the (One-Shot) Nash Equilibrium to Benefit the Large Player w/ the Intervention: Games of Full-Complementarities and Intervention Induces Beneficial Actions

Our objective below is to present a set of sufficient conditions under which the payoff of the large player in the one-shot Nash equilibrium in game G^2 is higher than that in game G^1. We first present a motivating example.

C.1. A Linear-Quadratic Example

Suppose that $\mathcal{A}_L = \mathcal{A}_S = [0, \infty)$. Consider game G^j and the following payoff structure of the large and the small players:

$$\begin{aligned} \pi^j(a_{S,i}, a_L, a_{S,A}) &= \left(\beta^j a_L + \alpha^j a_{S,A}\right) a_{S,i} - \frac{1}{2} a_{S,i}^2, \\ \Pi^j(a_L, a_{S,A}) &= \left(\tau^j + \gamma^j a_{S,A}\right) a_L - \frac{1}{2} a_L^2. \end{aligned}$$

where $\alpha^j, \beta^j, \gamma^j, \tau^j$ are strictly positive and $1 - \alpha^j > \beta^j \gamma^j$. The best-responses of the large player and an individual small player are:

$$\begin{aligned} a_L^*(a_{S,A}) &= \tau^j + \gamma^j a_{S,A} \\ a_{S,i}^*(a_L, a_{S,A}) &= \beta^j a_L + \alpha^j a_{S,A}. \end{aligned}$$

The symmetric best-response of the small players is:

$$a_{S,A}^*(a_L) = \frac{\beta^j}{1 - \alpha^j} a_L.$$

The Nash equilibrium actions of the players (with small players playing symmetric actions) and their equilibrium payoffs are as follows:

$$\begin{aligned} a_L^{NEj} &= \frac{\tau^j}{1 - \frac{\beta^j \gamma^j}{1 - \alpha^j}} \\ a_{S,i}^{NEj} = a_{S,A}^{NEj} &= \frac{\tau^j}{\frac{1 - \alpha^j}{\beta^j} - \gamma^j} \text{ (symmetric actions of small players)} \\ \Pi^{NEj} &= \frac{{\tau^j}^2}{2\left(1 - \frac{\beta^j \gamma^j}{1 - \alpha^j}\right)^2}, \\ \pi^{NEj} &= \frac{{\tau^j}^2}{2\left(\frac{1 - \alpha^j}{\beta^j} - \gamma^j\right)^2}. \end{aligned}$$

Observe that $(\Pi^{NE})^j$ is increasing in $\alpha^j, \beta^j, \gamma^j, \tau^j$.[16] The intervention G^2 involves higher values for the parameters, i.e., $\alpha^2 \geq \alpha^1$, $\beta^2 \geq \beta^1$, $\gamma^2 \geq \gamma^1$ and $\tau^2 \geq \tau^1$. Therefore,

$$\Pi_{NE}^2 \geq \Pi_{NE}^1.$$

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[16] In this example, a_L^{NEj}, $a_{S,A}^{NEj}$, Π^{NEj}, π^{NEj} are all increasing in $\alpha^j, \beta^j, \gamma^j, \tau^j$.

C.2. Main Model

For the purpose of illustration, we restrict attention to action spaces being a compact subset of the real line, i.e., $\mathcal{A}_L \subseteq \mathbb{R}, \mathcal{A}_S \subseteq \mathbb{R}$ and payoffs $\Pi^j(a_L, a_{S,A})$, $\pi^j(a_{S,i}, a_L, a_{S,A})$ continuous and differentiable throughout. For technical reasons, we assume that $\underline{a}_S := \min\{a_S : a_S \in \mathcal{A}_S\} > -\infty$. The key ideas below can be extended to a setting with action spaces from a finite set. Recall the definition of a seemingly-beneficial intervention from Assumption 2.2:

$$\text{for any } (a_L, a_{S,A}),\ \Pi^2(a_L, a_{S,A}) \geq \Pi^1(a_L, a_{S,A}).$$

Consider the following assumptions:

(a) (Strict Concavity)

$$\frac{\partial^2 \Pi^j}{\partial a_L^2} < 0 \text{ for all } a_{S,A}; \frac{\partial^2 \pi^j}{\partial a_{S,i}^2} < 0 \text{ for all } (a_L, a_{S,A}).$$

(b) (Full Complementarities)

$$\frac{\partial^2 \Pi^j}{\partial a_{S,A} \partial a_L} \geq 0 \text{ for all } a_{S,A}; \frac{\partial^2 \pi^j}{\partial a_L a_{S,i}} \geq 0 \text{ for all } a_{S,A},\ \frac{\partial^2 \pi^j}{\partial a_{S,A} a_{S,i}} \geq 0 \text{ for all } a_L.$$

(c) (Intervention Induces Beneficial Actions)

$$\frac{\partial \Pi^2}{\partial a_L} \geq \frac{\partial \Pi^1}{\partial a_L} \text{ for all } a_{S,A}; \frac{\partial \pi^2}{\partial a_{S,i}} \geq \frac{\partial \pi^1}{\partial a_{S,i}} \text{ for all } (a_L, a_{S,A}).$$

(d) (Large Player Prefers Higher Average Play of the Small Players)

$$\frac{\partial \Pi^j}{\partial a_{S,A}} \geq 0 \text{ for all } a_L.$$

We explain these conditions.

- Part (a) implies that Π^j (resp., π^j) is strictly concave in a_L (resp., in $a_{S,i}$) for any $a_{S,A}$ (resp., $a_L, a_{S,A}$). We assume strict concavity to ensure uniqueness of the best-response functions. Our main result holds with weak concavity, but involves refining the best-response correspondences and the resulting set of equilibria. Let $a^j_{L,B}(a_{S,A})$ (resp., $a^j_{S,B}(a_L, a_{S,A})$) denote the best responses of the large player (resp., an individual small player) in game G^j.

$$a^j_{L,B}(a_{S,A}) = \arg\max_{a_L \in \mathcal{A}_L} \Pi^j(a_L, a_{S,A}) \quad \text{(resp., } a^j_{S,B}(a_L, a_{S,A}) = \arg\max_{a_S \in \mathcal{A}_S} \pi^j(a_L, a_S, a_{S,A}))$$

- Part (b) shows that the marginal increase in the payoff of the large player from a higher action is increasing in the average of the small players' actions (resp., the marginal increase in the payoff of an individual small player from a higher action is increasing in the large player's action and the average of all the small players' actions). Therefore,

$$\frac{da^j_{L,B}(a_{S,A})}{da_{S,A}} \geq 0 \quad \text{(resp., } \frac{\partial a^j_{S,B}(a_L, a_{S,A})}{\partial a_L} \geq 0 \text{ and } \frac{\partial a^j_{S,B}(a_L, a_{S,A})}{\partial a_{S,A}} \geq 0).$$

  Taken together, we have that game G^j corresponds to a game of full complementarities, i.e., the best-response of every player is increasing in the actions of the other players.

- Part (c) shows that for any $a_{S,A}$ (resp., $a_L, a_{S,A}$), the best-response functions are higher in game G^2 than in G^1, i.e.,

$$a^2_{L,B}(a_{S,A}) \geq a^1_{L,B}(a_{S,A}) \quad (\text{resp., } a^2_{S,B}(a_L, a_{S,A}) \geq a^1_{S,B}(a_L, a_{S,A})). \tag{C.1}$$

  Taken together, we have that the intervention induces higher (or beneficial) actions from all the players.

- Part (d) above is a special case of Assumption 2.3, applied to real-valued action spaces, i.e., for any a_L, the large player's payoff in G^j is increasing the average play $a_{S,A}$ of the small players.

Furthermore, we assume the following assumptions that lead to the uniqueness of the symmetric best-response of the small players:

(e) $a^1_{S,B}(a_L, \underline{a}_S) > \underline{a}_S$ for all a_L.

(f) There exists $r < 1$ such that $\frac{\partial a^j_{S,B}(a_L, a_S)}{\partial a_S} \leq r$.

Part (e) shows that the best-response of an individual small player at $a_{S,A} = \underline{a}_S$ is strictly larger than $\underline{a}_S$. Parts (e) and (f) are technical conditions that are required for the uniqueness of the symmetric best-response of the small players explained below. Consider game G^j and any $a_L \in \mathcal{A}_L$. The symmetric best response of the small players, $a^j_{S,EQ}(a_L)$, solves

$$a_S = a^j_{S,B}(a_L, a_S). \tag{C.2}$$

From (e) and (f), it follows that $a^j_{S,EQ}(a_L)$ is unique; from (b), it follows that $a^j_{S,EQ}(a_L)$ is increasing in a_L. Combining this with (C.1), we have that

$$a^2_{S,EQ}(a_L) \geq a^1_{S,EQ}(a_L). \tag{C.3}$$

(C.1) and (C.3) together show that the best-response function of the large player and the symmetric best-response of the small players are higher in G^2 than in G^1. (C.3) above is identical to Assumption 2.4 if the set of symmetric best-response for any a_L is singleton. Further, from (e), we have that

$$a^j_{S,EQ}(a_L) > \underline{a}_S \text{ for all } a_L. \tag{C.4}$$

To see this, observe that for any a_L, $a_S = \underline{a}_S$ does not solve (C.2).

Consider a (symmetric) Nash equilibrium of game G^j, denoted by $NE^j = (a^{NE^j}_L, a^{NE^j}_{S,A})$. From the definition of a Nash equilibrium, NE^j satisfies the following:

$$a^j_{S,EQ}(a^{NE^j}_L) = a^{NE^j}_{S,A} \text{ and } a_{L,B}(a^{NE^j}_{S,A}) = a^{NE^j}_L. \tag{C.5}$$

Finally, we assume:

(g) game G^j admits a unique (symmetric) Nash equilibrium, denoted by NE^j.

Lemma C.1. *The equilibrium actions of all players in game G^2 are higher than those in game G^1, i.e.,*

$$a^{NE^2}_L \geq a^{NE^1}_L \text{ and } a^{NE^2}_{S,A} \geq a^{NE^1}_{S,A}.$$

Proof: From (C.1), (C.3) and (C.5), it suffices to show that $a_{S,A}^{NE^2} \geq a_{S,A}^{NE^1}$. From (C.5), $a_{S,A}^{NE^j}$ solves

$$a_S = a^j_{S,EQ}(a^j_{L,B}(a_S)),$$

Recall from above that $a^j_{L,B}(\cdot)$ and $a^j_{S,EQ}(\cdot)$ are increasing in their arguments. Define $\phi^j(a_S) = a^j_{S,EQ}(a^j_{L,B}(a_S)) - a_S$. Using (C.1) and (C.3), we have that

$$\phi^2(a_S) \geq \phi^1(a_S). \tag{C.6}$$

From (g), since NE^j is unique, $\phi^j(\cdot)$ has a unique root at $a_S = a_{S,A}^{NE^j}$. From (C.4), we have that $\phi^1(\underline{a}_S) > 0$. Therefore, for $a_S \in [\underline{a}_S, a_{S,A}^{NE^1})$, we have that $\phi^1(a_S) > 0$. Combining this observation with (C.6), we have the required result. Q.E.D.

Theorem C.1. *The equilibrium payoff of the large player in game G^2 is higher than that in game G^1, i.e.,*

$$\Pi^2(NE^2) \geq \Pi^1(NE^1).$$

Proof: From Assumption 2.2, we have:

$$\Pi^2(NE^1) \geq \Pi^1(NE^1).$$

From Lemma C.1 and part (a), we have:

$$\Pi^2(a_L^{NE^1}, a_{S,A}^{NE^2}) \geq \Pi^2(NE^1)$$

From the definition of $a^2_{L,B}(\cdot)$, we have:

$$\Pi^2(NE^2) \geq \Pi^2(a_L^{NE^1}, a_{S,A}^{NE^2})$$

Combining the three inequalities above, we have the required result. Q.E.D.

Remark C.1. Our main model in Section C.2 assumes that G^j, $j \in \{1,2\}$ are games of full complementarities and the intervention induces beneficial actions. In such games, the payoff of the large player in the symmetric, one-shot Nash equilibrium in G^2 exceeds that in G^1. In Section C.1, we provide an example that satisfies the proposed conditions (a)-(g) and the thus, the intervention leads to higher profits to the large player in the one-shot (symmetric) Nash equilibrium. Recall our motivating examples in Section 3 considers outcomes where the payoff of the large player in the symmetric one-shot Nash equilibrium in G^1 exceeds that in G^2.

- In Section 3.1.1, recall from (3.2) that the station's best-response is decreasing in the average quality of the milk produced by the farmers. That is, the large player's best-response is decreasing in the average of the small players' actions; thus, this is not a game of full complementarities.
- In Section 3.2.1, the best-response of an individual small player and the best-response of the large player are decreasing in the average of the small players' actions (i.e., this is not a game of full complementarities).

That is, these examples do not satisfy the assumptions of the model in Section C.2. Hence, the one-shot games lead to a lower equilibrium payoff to the large player.