An Economic Analysis of Agricultural Support Prices in Developing Economies

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Several developing countries have adopted the Guaranteed Support Price (GSP) scheme to support their farmers and underprivileged population. Through this scheme, the government, operating under a budget, procures a crop from farmers at a guaranteed (and attractive) price, announced ahead of the selling season, and then distributes the procured amount to the underprivileged segment of the population at a subsidized price. The goal of this scheme is twofold: (a) as a supply-side incentive, to ensure high output from the farmers, and (b) as a demand-side provisioning tool, to subsidize the consumption needs of the poor.

We analyze a Stackelberg game between a social planner and a population of farmers. We model the strategic behavior of the farmers and the consuming population, characterize the equilibrium market outcome (i.e., the production decisions of the farmers and their selling decisions to the government and in the open market, the consumption decisions by the Above-Poverty-Line (APL) and Below-Poverty-Line (BPL) consumers), and compare the equilibrium welfare of each segment with that under two benchmarks: (a) the absence of any intervention, and (b) the Direct Benefit Transfer (DBT) scheme, where the social planner simply distributes the budget among the BPL consumers. Two key economic forces – the poorness of the BPL consumers (a demand-side force), and yield uncertainty (a supply-side force) – act as impediments to high production by the farmers and consumption by the BPL consumers. If the poorness of the BPL consumers is extreme, then the GSP scheme leads to a strict improvement in the production by the farmers, consumption by the BPL consumers, and the social planner's surplus. If yield uncertainty is dominant, then the social planner can use the GSP scheme as a mechanism to divide his budget in any proportion to improve the surplus of the BPL consumers and the farmers; the desired split is achieved by setting an appropriate support price. We also analyze the two schemes in the presence of leakages – wealth leakage under DBT and grain pilferage under GSP. For reasonable values of the leakages, we show that the production by the farmers is strictly higher under the GSP scheme. If the extent of wealth leakage is sufficiently large, then the social planner's surplus under GSP is strictly higher than that under DBT. The GSP scheme also allows the social planner to maintain a reserve stock of food grains, thus improving food security.

Key words: Support Prices; Agricultural Operations; Operations in Developing Economies.

1. Introduction

Due to its significant contribution to the gross domestic product and its critical role in shaping the livelihood of a majority of the population, the agricultural sector in many developing economies attracts substantial support from the government. Among the various governmental schemes that support agriculture, *Guaranteed Support Prices* (GSPs; also called *Minimum Support Prices*, or simply, *support prices*) have been adopted by many developing nations, including Bangladesh, Brazil, China, India, Pakistan, Thailand, and Turkey. A guaranteed support price for an agricultural crop is an attractive price at which the government promises to purchase that crop from farmers, regardless of its market price. We first discuss the motivation behind the GSP scheme and the factors that influence the government's decision to offer such a price for a particular crop.

To support our discussion with data throughout this introduction, we will use India as an example of a developing nation. India first introduced its support-price program in the mid 1960s, during the green revolution, as a *supply-side* incentive: An attractive support price for a crop protects the farmers from the adverse effects on its market price due to overproduction and naturally incentivizes them to increase input effort. Over the years, the GSP scheme has been widely credited for the consistent increase in productivity as well as the increase in farming inputs; see, e.g., USDA Foreign Agricultural Service (2014), Directorate of Economics and Statistics of India (2014).

Driven by changes in economic and demographic factors, a secondary purpose of a support price has been as a *demand-side* provisioning tool: The increase in farming effort, engineered via a support price, results in high production, of which the government procures a significant amount and distributes it to the economically weak population at a nominal price via a network of ration shops (Government of India 2010, SMC Investments 2010, Parikh and Singh 2007, Planning Commission of India 2001).

We now discuss the primary factors that influence the value of the support price for a crop. Typically, support prices are determined on an annual basis. In India, the Commission for Agricultural Costs and Prices recommends the values of the support prices for various crops to the government. The recommendation for a specific crop primarily depends on its (a) demand and supply, (b) cost of production, and (c) yield uncertainty; see, e.g., Ministry of Agriculture, Government of India (2018), Government of India (2010), Kang (2012).

(a) **Demand-Supply Gap**: Consider a crop that experiences a significantly higher market demand than its supply. The insufficient supply naturally drives the market price high, resulting in the poor finding it difficult to afford the crop at an elevated price. Since the below-poverty-line (BPL) population in developing nations is significant, the government may intervene by offering a support price for this crop to increase its supply and, thereby, its consumption by the population.

(*b*) **Production Cost**: An increase in the cost of farming inputs directly increases the cost of production. One reason for the increase in input costs is the progressive reduction in the average size of a farm. Small farmers are unable to go for mechanization of farming due to physical limitations and, therefore, have to depend on manual labor, which is becoming increasingly scarce and expensive (Kumaraswamy 2012). As production cost increases, it is natural to expect a decrease in the total input effort, resulting in a reduction in the total production and, ultimately, the food-grains available for consumption. Therefore, to elevate the level of input effort by assuring farmers of "good" income that can compensate for the increase in production cost, the government may offer an attractive support price.

(*c*) **Crop-Yield Uncertainty**: Yield uncertainty increases fluctuation in supply (i.e., food-grain production), resulting in unstable market prices and, hence, unstable revenue for farmers. Thus, the government again has an incentive to offer a support price and avoid socially undesirable outcomes by safeguarding farmers from adverse price fluctuations (Gomez-Limon et al. 2002).

It should be clear that the value of the support price for a crop should consider the combined influence of these factors. Our goal in this paper is to offer analytically-supported insights on several fundamental aspects of the GSP scheme, including its impact on social welfare. Clearly, this analysis should anticipate the operational decisions of the farmers, i.e., their production decisions and their selling decisions to the government and in the open market, in response to a potential support price. Accordingly, we characterize the equilibrium decisions of the farmers enroute to assessing the impact on social welfare. This analysis then allows us to understand how the effectiveness of the GSP scheme, as a supply-side incentive and as a demand-side provisioning tool, is affected by the budget and the characteristics of the population (e.g., the "poorness" of the BPL population), and by yield uncertainty.

In addition to the above factors, our analysis captures other real-world characteristics:

(*d*) **"Small" Farmers**: A majority of farmers in developing countries are small-holders, with each owning or cultivating a small piece of land (on average, less that 2.0 hectares). Accordingly, farmers are assumed to be *price-takers*, i.e., the actions of an individual farmer do not influence the market price.

(*e*) **Multiple Selling Channels for Farmers and a Heterogeneous Consuming Population**: In the presence of the Guaranteed Support Price scheme for a crop, a farmer has access to two different outlets to sell his produce: (i) selling the crop to the government at the support price, or (ii) selling in the open market at the market price. In the open market, we consider two different consumer segments: (A) the Above-Poverty-Line or APL consumers, and (B) the Below-Poverty-Line or BPL (or simply "poor") consumers. Our analysis captures both the extent of the "poorness" of the BPL consumers and the relative size of this segment.

(f) **Limited Budget of the Government**: The support-price program for a crop – buying the crop from farmers at the GSP and providing it to the BPL population at a nominal price via a country-wide public distribution system (PDS) of ration shops – is financed by a limited budget, which the government determines annually based on the country's economical, demographical, and political environment. In India, for instance, the combined budget for the support-price programs of a total of 26 crops was $16 billion in 2016; this is then partitioned into budgets for individual crops (The Wall Street Journal 2016). Accordingly, our analysis assumes a limited budget for the support-price program of a crop.

1.1. The Direct Benefit Transfer (DBT) Scheme

Besides the GSP scheme, other schemes that directly transfer monetary subsidies, i.e., cash benefits, to the individual beneficiaries (through their bank accounts) have also received increasing patronage in recent years. For instance, in India, a program to pilot the Direct Benefit Transfer (DBT) scheme was launched in January, 2013, in an effort to increase transparency, eliminate pilferage of funds, and streamline existing processes of government delivery across welfare schemes. By directly transferring a subsidy to the bank accounts of the intended beneficiaries, the government is able to eliminate losses that arise from intermediaries or middlemen. Therefore, the government is better able to realize its objectives, namely *"... simpler and faster flow of information/funds and to ensure accurate targeting of the beneficiaries, de-duplication and reduction of fraud ..."*; see Direct Benefit Transfer (2018). Moreover, the DBT scheme would alleviate costly storage and distribution costs that are associated with the GSP scheme.

Despite its potential benefits, several operational factors are critical to the success of the DBT scheme. In particular, the identification and digitization of the database of beneficiaries, the opening of their bank accounts, the enrollment of unique identification (also known as Aadhaar in India) of the beneficiaries and their seeding with the beneficiary database and their bank accounts, and the last-mile delivery of the benefits via banking correspondents (as an alternative to brick and mortar banks) are key to ensuring its success. In a recent study by Muralidharan et al. (2017) on the pilot implementation of the DBT scheme in certain states, the authors find that most beneficiaries take 1.8 – 3.5 times as much time to withdraw funds from a bank and go to the market to buy grains than it took to get them from the ration shops that distribute food grains under the GSP scheme. Inefficiencies in accurate identification and digitization of the beneficiary database has led to several beneficiaries receiving no subsidy, and in many cases, erratic cash transfers. Besides, the beneficiaries found the cash to be inadequate to buy the same quantity of food grains that they would have received through the ration shops. They conclude that, without assured and timely subsidy payments, consumers are reluctant, and in some cases, unable to buy food grains at the market rate. Producers, on their part, are also concerned about the eventual sale of their produce. Surveys by major media outlets, such as India Today (2018) and The Hindu (2016), report consistent stories of the shortcomings of the DBT scheme. Besides, other social factors, such as abuse of the subsidy for other activities, including alcohol, have also affected the success of the DBT scheme; see The Wire (2018).

1.2. In-Kind vs. In-Cash Subsidies

In-kind subsidies (e.g., the GSP scheme) have been historically important and continue to remain prevalent in developing economies (Gadenne et al. 2017). Honorati et al. (2015) estimate that 44% of beneficiaries of social safety net programs around the world receive in-kind transfers. However, an increasing body of research in economics and public policy advocates for cash as the preferred form of subsidy (e.g., the DBT scheme). The proponents of cash transfers (conditional and unconditional) argue that they give the poor the flexibility to choose the best opportunities, rather than having the opportunities chosen for them. Blattman et al. (2017) state that "governments in emerging markets have begun to shift from expensive, regressive and distortionary subsidies of basic commodities such as food or fuels and instead are giving cash to the poor". On the contrary, Gadenne et al. (2017) find that in-kind transfers provide insurance against commodity price risk. This is especially prevalent in developing countries where markets are poorly integrated. Our analysis in Section 8 complements Gadenne et al. (2017) in informing this debate about the merits of in-kind subsidies.

1.3. Summary of Results

Our analysis offers analytically-supported clarity on the impact of the GSP scheme. We expound on two key economic forces – the poorness of the BPL consumers (a demand-side friction) and yield uncertainty (a supply-side friction) – that act as impediments to high production by the farmers, and consumption by the poor consumers.

- If the poorness of the BPL consumers is extreme, then the GSP scheme helps in improving the production of the farmers and the consumption by the BPL consumers. In this case, although the surplus of the APL consumers is lower under the GSP scheme (relative to its absence), the surplus of the farmers, the BPL consumers, and the social planner are all higher. Further, the equilibrium market outcome and the social planner's surplus are identical to that under the DBT scheme.

- If yield uncertainty is a dominant criterion in determining the equilibrium effort of the farmers, then the surplus of the social planner under the equilibrium GSP is identical to that under the DBT scheme, which in turn is identical to that under no intervention. Nevertheless, under the GSP scheme, the social planner can divide his budget in *any* desired proportion to improve the surplus of the farmers and the BPL consumers by using an appropriate support price.

We also consider a weighted objective of the social planner and demonstrate how different weights on the stakeholder's surplus affect the equilibrium support price. A higher weight on the farmers' surplus (resp., surplus of the BPL consumers) leads to an increase (resp., decrease) in the support price.

We identify real-world situations under which the GSP scheme outperforms the DBT scheme:

- In the presence of leakages (or losses) – namely, *wealth leakage* in the DBT scheme and *grain pilferage* in the GSP scheme – the total production by the farmers is strictly higher under the GSP scheme than that under the DBT scheme for reasonable values of the leakages. If the wealth leakage (or hassles in accessing the cash subsidy) under the DBT scheme is sufficiently high, the social planner's surplus under the GSP scheme exceeds that under the DBT scheme.
- Beyond the twin goals of improving production (by the farmers) and consumption (by the BPL consumers), a third goal of the GSP scheme is in improving food security by mitigating the adverse effects of yield uncertainty. The GSP scheme allows the social planner to maintain a reserve stock of food grains. If the marginal value from maintaining a reserve stock is sufficiently high, then the social planner's surplus under the GSP scheme is higher than that under the DBT scheme.

1.4. Organization of the Paper

The rest of the paper is organized as follows. We review the related literature in Section 2, introduce our model in Section 3, and analyze the market outcome under (a) the absence of any intervention in Section 4, (b) the DBT scheme in Section 5 and (c) the GSP scheme in Section 6. We consider a weighted objective of the social planner (where he weighs the surplus of the farmers and the BPL consumers more than that of the APL consumers and the unused budget) in Section 7. We conduct numerical experiments, and analyze the role of leakages and their effect on the market outcomes in the respective schemes in Section 8. Finally, we conclude in Section 9.

2. Related Literature

While GSPs have been investigated in the agricultural economics literature, a majority of these studies focus on strategic decisions such as the impact of price support on international trade and the number of firms in the industry, and the need for market adjustments due to support prices. Studies that are representative of this line of research include Fox (1956), Dantwala (1967), Spitze (1978), Sjoquist (1979), Gulati and Sharma (1994), Food and Agriculture Organization of United Nations (2001), Cummings Jr et al. (2006), Josling et al. (2010). In contrast, as discussed above, our aim is to characterize operational decisions of the farmers and the government, and understand the welfare implications of the GSP scheme for the farmers and the consuming population. Besides, we also analyze the DBT scheme and compare the market outcomes under both the GSP and DBT schemes and that under no intervention.

Close to our work, Kazaz et al. (2016) analyze interventions to improve production of artemisenin (used in the malaria-medicine supply chain) under demand and supply uncertainty, and find that support prices lead to greater production by the farmers. However, their setting consists only of the open-market with exogenous demand. In our setting, a budget-constrained social planner announces a support price, procures grains from farmers, and distributes them among the BPL consumers (one segment of the consumer population). Besides, farmers can also sell in the open-market, which is accessible to both APL and BPL consumers. Further, we model the strategic behavior of the APL and BPL consumers and the farmers explicitly.

Motivated by schemes in developed countries, Alizamir et al. (2018) consider two subsidy programs: (a) Price Loss Coverage (PLC) and (b) Agricultural Risk Coverage (ARC). Under PLC, the government offers farmers a subsidy if the market price falls below a reference price, while under ARC, the farmers receive a subsidy if their revenue falls below a threshold. They model competition among a finite population of farmers who sell in the open-market and compare the equilibrium market outcome under both these schemes to that under no intervention. Contrary to conventional wisdom, they find that the producer welfare, consumer welfare, and social welfare, can all be higher under PLC than under ARC. Our setting differs from theirs in many aspects. We consider competition among infinitesimally small farmers who can sell to the social planner as well as in the open-market. The social planner distributes the procured quantity among the BPL consumers, while both the APL and the BPL consumers can access the open market.

Relatedly, Akkaya et al. (2016) study the impact of interventions such as price and cost support under public and private information about government budget on the social welfare. However, in their context, a support price is essentially a price floor that the government announces before the sowing season and pays the farmers the difference between the support price and the open-market price for each unit sold in the open-market. While we consider the interactions among strategic farmers and consumers, in a recent paper, Hu et al. (2019) study the market outcome under a mix of myopic and strategic farmers. They find that a small fraction of strategic farmers can stabilize fluctuating market prices. Chintapalli and Tang (2018) consider farmers' crop-planting decisions when the social planner offers a GSP for two crops. Akin to Akkaya et al. (2016), a support price in their context is "credit-based", i.e., the government does not procure any quantity from the farmers; rather, it pays the farmers the difference between the support and the market price for each unit sold in the market. They find that farmers do not internalize the externality they impose on other farmers in their planting decisions and therefore, vis-a-vis a setting where the social planner chooses the planting decisions, the decentralized planting decisions of the farmers leads to a loss of producer, consumer, and social welfare. It is worth mentioning that both Hu et al. (2019) and Chintapalli and Tang (2018) model the strategic behavior of producers and consumers but do not consider the effect of yield uncertainty.

The GSP scheme is related to a *price-floor*, a concept well-studied in microeconomics; see Varian (1992). In Section 6, we comment on the impact of the GSP scheme and the price floor on the decisions of the different stakeholders as well as on their welfare.

Our work caters to the growing interest in the Operations Management (OM) community on agricultural operations; some recent examples include Huh and Lall (2013) (crop rotation); Dawande et al. (2013) (distribution of surface water between farmers); Chen and Tang (2015), Parker et al. (2016), Tang et al. (2015) (provision of valuable information to farmers); An et al. (2015) (farmer aggregation); Federgruen et al. (2019) (contract farming); Levi et al. (2018) (adulteration in farming supply chains).

A support price is an incentive aimed at improving production. The design of incentives that enable firms to elicit favorable decisions from supply-chain partners is a well-established research area in OM; see, e.g., the survey by Cachon (2003). There is also a growing interest in the OM community in analyzing incentives aimed at improving social welfare and understanding micro-level decisions that consumers make in response to such incentives; some recent examples include Avci et al. (2014), Cohen et al. (2015) (subsidy programs for adoption of electric vehicles and their environmental impact); Mu et al. (2015) (programs to reduce adulteration and improve quality in milk supply chains); Lobel and Perakis (2011) (subsidy programs for adoption of solar panels); Atasu et al. (2009) (programs to promote product recycling); Raz and Ovchinnikov (2015) (government intervention mechanisms for public interest goods).

3. The Model

Consider a social planner (the government), a homogenous farming population of size[1] n that produces a crop, and a consuming population consisting of two segments: (a) the Above-Poverty-Line, or APL consumers of size M, and (b) the Below-Poverty-Line, or BPL (poor) consumers of size kM. The sequence of events is as follows:

Stage 1: Ahead of the sowing season, the social planner, fuelled by a budget B for the GSP scheme, announces the per-unit GSP p_g for the crop.

Stage 2: Each farmer decides his input effort based on the announced GSP, the cost of production, and the distribution of yield uncertainty. Let q_e denote the input effort of a representative farmer. His production cost is modeled as follows: For a fixed q_e, the farmer incurs a production cost of αq_e^2, where α is the production-cost parameter of the farming population. Several papers in the OM literature have modeled farming cost as a quadratic function of the farmer's effort; see, e.g., Alizamir et al. (2018).

Stage 3: The production yield uncertainty is realized: Let γ denote the realized yield. For each farmer, the output corresponding to an input effort of q_e is γq_e.

Stage 4: Each farmer decides the quantities to be sold to the social planner and in the open-market, where the per-unit market price p_m is simultaneously realized. Let q_g (resp., q_m) denote the quantity sold by the farmer to the social planner (resp., in the open-market). The social planner distributes some or all[2] of the procured quantity among the BPL consumers at no cost.[3] The BPL consumers have the option of reselling some of the quantity that they receive from the social planner back into the open market.

The objective of each farmer is to maximize his expected profit while the objective of the social planner is to maximize the expected social welfare. Our main notation is defined in Table 1 in Appendix A. The demand model for the APL and the BPL consumers is presented in Section 3.1 while the objective of the farmers (the supply model) is presented in Section 3.2. Finally, the social planner's objective is presented in Section 3.3.

[1] Throughout this paper, by size, we refer to the mass of a segment.

[2] For simplicity, we assume here that the entire quantity purchased by the social planner is made available for consumption by the BPL population. In reality, the social planner may distribute a fraction of the procured quantity among the BPL consumers and use the rest for other purposes, e.g., to maintain a buffer stock of foodgrains (for instance, in India, there are explicit stocking norms such as the minimum buffer stock which are mandated by the Cabinet Committee on Economic Affairs (CCEA) on a quarterly basis. This is done for operational reasons, such as meeting monthly distributional requirements, or for food security reasons, in events of unexpected shortfall in procurement; see Food Corporation of India (2017)). Our analysis can be easily modified in the absence of this assumption, without changing the nature of the results and insights. We discuss this assumption further at the end of Section 3.3.

[3] In practice, a nominal unit price is charged from the BPL consumers; we ignore this purely for simplicity of exposition. We further discuss this assumption in Section 3.3.

3.1. The Demand Model

Recall the two homogenous consumer segments – APL (size M) and BPL (size kM). The maximum consumption of any consumer in either segment is normalized to 1. Thus, the aggregate consumption by the APL consumers (resp., the BPL consumers) is at most M (resp., kM). We present the consumer utility models for APL and BPL consumers below.

APL Consumers Suppose w_{APL} ($>> 1$) denotes the wealth of the APL consumers. At a consumption level of ξ, the additional consumption utility to a consumer from the incremental consumption of an infinitesimal quantity $d\xi$ is $(1-\xi)d\xi$. Thus, the marginal consumption utility for a consumer monotonically decreases from 1 (at the minimum consumption level of 0) to 0 (at the maximum consumption level of 1). Corresponding to a consumption quantity $q \in [0,1]$, the net utility derived by an APL consumer is

$$u_{APL}(q) = \underbrace{\int_0^q (1-\xi)d\xi}_{=u_C(q)} + (w_{APL} - p_m q),$$

where $u_C(q) = \int_0^q (1-\xi)d\xi$ is the utility derived from consumption and p_m is the market price. The utility-maximizing quantity consumed by an APL consumer from the open-market at a market price $p_m \in [0,1]$ is

$$q^*_{APL} = \arg\max_{q\geq 0}\ u_{APL}(q) = \arg\max_{q\geq 0}\ \left[\int_0^q (1-\xi)d\xi + (w_{APL} - p_m q)\right] \tag{1}$$
$$= (1-p_m) \tag{2}$$

BPL Consumers The BPL consumers also have access to the open market. The consumption utility model for the BPL consumers is identical to that of the APL consumers', with two differences: (a) these consumers are assumed to be budget-constrained; i.e., they have lower wealth relative to the APL consumers: Let b (<1) denote the wealth of a BPL consumer; and (b) the social planner supplements these consumers with additional quantity for consumption: Let q_S denote the quantity that a BPL consumer receives from the social planner.

We allow for the possibility that the BPL consumers may resell a fraction of the quantity they receive from the social planner back in the open market. Suppose the social planner provides each BPL consumer with a quantity q_S. Then, one of the following outcomes hold:

(a) The market price is smaller than the marginal consumption utility at q_S, i.e., $p_m \leq 1-q_S$, or
(b) the market price is strictly larger than the marginal consumption utility at q_S, i.e., $p_m > 1-q_S$.

If the market price is smaller than the marginal consumption utility at q_S (i.e., (a) holds), then the BPL consumers do not find it profitable to sell any quantity they receive from the social planner back into the open-market (rather, they may purchase some quantity from the open-market). On the other hand, if the market-price is strictly higher than the marginal consumption utility at q_S (i.e., (b) holds), then the BPL consumers do not purchase from the open-market; instead, they will find it profitable to sell a fraction of the quantity they receive from the social planner back into the open-market. Combining these two cases, it is straightforward that a BPL consumer either sells in the open-market, or purchases from the open-market, but not both. Let q denote the quantity a BPL consumer consumes from the open market: If the BPL consumer sells in the open-market, then $q \leq 0$; if he purchases from the open market, then $q \geq 0$. The utility-maximizing quantity consumed by a BPL consumer from the open-market at a market price $p_m \in [0,1]$ is

$$q^*_{BPL} = \underset{q\in\left[-q_S,\frac{b}{p_m}\right]}{\arg\max}\ u_{BPL}(q) = \underset{q\in\left[-q_S,\frac{b}{p_m}\right]}{\arg\max}\left[\underbrace{\int_0^{q+q_S}(1-\xi)d\xi}_{=u_C(q+q_S)} + (b-p_m q)\right]. \tag{3}$$

Thus, we have

$$q^*_{BPL} = \min\left\{(1-q_S-p_m), \frac{b}{p_m}\right\}. \tag{4}$$

Observe that the quantity $(1-q_S-p_m)$ can be negative, if the market price p_m exceeds $1-q_S$ (the marginal consumption utility at q_S). Since all BPL consumers are homogeneous, we assume that the social planner supplements each consumer with the same quantity q_S and they consume the same quantity q^*_{BPL} from the open market.

Combining (2) and (4), the aggregate consumer demand at a market price $p_m \in [0,1]$ can be written as:

$$D(p_m) = Mq^*_{APL} + kMq^*_{BPL} = M(1-p_m) + kM\min\left\{(1-q_S-p_m), \frac{b}{p_m}\right\}. \tag{5}$$

Alternately, given a certain quantity, say Q, available to be sold in the open-market, the market price p_m is determined by the above equation, i.e., $p_m = D^{-1}(Q)$. We restrict attention to small values of b to reflect the "poorness" of the BPL consumers – we will make this precise in Section 4.

3.2. The Supply Model

We now formulate the decision problems of the farmers. Recall that farmers are assumed to be price-takers, i.e., the actions of an individual farmer do not affect the market outcome in any realistic way. We explicitly model the interactions among strategic farmers. Specifically, farmers face competition from other farmers in their selling decisions in the open-market and to the social planner. Moreover, they anticipate the consequences of their selling decisions to the social planner: First, they anticipate a downward shift in the BPL consumers' demand curves in the open market (that is proportional to the quantity sold by the farmers to the social planner). Second, they anticipate that the BPL consumers may resell some of the quantity they receive from the social planner back in the open-market. Let $\hat{p}_m(\gamma)$ denote the belief that a farmer has about the market price (as a function of the realized yield). The homogeneity of farmers allows us to make two assumptions:

ASSUMPTION 3.1. *All farmers hold the same belief $\hat{p}_m(\gamma)$ about the market price, and their production and selling decisions are identical.*

Therefore, we assume that farmers hold rational beliefs, i.e., the actions taken by the farmers given their beliefs lead to an outcome that is consistent with their beliefs. This is a standard assumption in the literature, consistent with the theory of rational expectations. Further, they take identical actions, i.e., they exert an effort q_e, and sell q_m (resp., q_g) in the open-market (resp., to the social planner).[4]

Consider a representative farmer, who derives revenue from two sources:

(a) by selling to the social planner, and

(b) by selling in the open market.

For any support price $p_g \geq 0$ chosen by the social planner, the farmer's belief $\hat{p}_m(\gamma)$ about the market price, the effort q_e chosen by the farmer in the second stage, and the yield γ chosen by nature in the third-stage, the fourth-stage optimization problem is as follows: The farmer decides q_m and q_g, the quantity of produce sold in the open market and to the social planner, respectively. Since costs are sunk in the second stage, it suffices to maximize his revenues in the fourth stage. That is,

$$\left.\begin{aligned} & r^{\gamma}(q_e) = \max_{q_m, q_g} \quad [\hat{p}_m(\gamma) q_m + p_g q_g] \\ & \text{subject to:} \\ & \qquad\qquad q_m + q_g \leq \gamma q_e, \\ & \qquad\qquad p_g q_g \ \leq \tfrac{B}{n}, \\ & \qquad\qquad (q_m, q_g) \geq 0. \end{aligned}\right\} \quad \texttt{Problem } \mathtt{P}_{\mathtt{f}}^2$$

Denote the optimal values of q_m (resp., q_g) obtained from $\texttt{Problem P}_{\mathtt{f}}^2$ by q_m^* (resp., q_g^*). Indeed, q_m^* and q_g^* depend on the realized yield γ; for notational simplicity, we avoid stating this dependence explicitly. The second constraint, $p_g q_g \leq \frac{B}{n}$ (in the set of constraints above) pertains to the

[4] An alternative is to consider asymmetric pure strategies, where each farmer either sells to the social planner or in the open market. In such a case, while the equilibrium effort of all the farmers is identical, we solve for the proportion of farmers that sell in the open market and to the social planner. It can be easily seen that this does not alter the quantity procured by the social planner and made available in the open market. Consequently, the market price, and therefore, the market outcome, are identical.

maximum quantity that the social planner procures from an individual farmer – we assume that the social planner rations his budget equally among the farmers. We then write the second-stage optimization problem as follows:

$$\left.\max_{q_e \geq 0} \;\; \pi_f(q_e) = -\alpha q_e^2 + \mathbb{E}_\gamma\left[r^\gamma(q_e)\right]. \;\;\right\} \qquad \texttt{Problem } \mathtt{P}_{\mathtt{f}}^{\mathtt{1}}$$

Let q_e^* denote the optimal value of q_e obtained from $\texttt{Problem } \mathtt{P}_{\mathtt{f}}^{\mathtt{1}}$.

3.3. Objective of the Social Planner

The objective function Π_S of the social planner is the sum total of the surplus derived by each segment of the population, and consists of the following components: (a) the "consumer surplus" derived by the APL and the BPL consumers from consumption, $Mu_{APL} + kMu_{BPL}$, (b) the "producer surplus" derived by the farmers, $n\pi_f$ and (c) unused budget of the social planner. Thus, the social planner's problem is as follows:[5]

$$\max_{p_g \geq 0} \; \Pi_S = \mathbb{E}_\gamma \left[Mu_{APL} + kMu_{BPL} + n\pi_f + (B - np_g q_g) \right]. \tag{6}$$

Observe from (1) and (3) that the terms w_{APL} and b constitute transfers from the consumers to the farmers. Substituting for the equilibrium consumption quantities of the APL and BPL consumers from (2) and (4) in (6), the social planner's surplus can be written as:

$$\begin{aligned}\Pi_S = {} & (Mw_{APL} + kMb + B) + \\ & \mathbb{E}_\gamma \left[M \int_0^{q^*_{APL}} (1 - p_m(\gamma) - \xi)\, d\xi + kM \left(\int_0^{q_S + q^*_{BPL}} (1 - \xi)\, d\xi - p_m(\gamma) q^*_{BPL} \right) - np_g q_g \right] + n\pi_f(q_e^*)\end{aligned}$$

The first term is the total wealth across all stakeholders and is a constant. Therefore, it is sufficient to consider the remaining terms in the social planner's objective. Further, all monetary payments consist of transfers from one agent to another. Therefore, we can rewrite the social planner's surplus as follows:

$$\Pi_S = (Mw_{APL} + kMb + B) + \mathbb{E}_\gamma \left[M \int_0^{q^*_{APL}} (1 - \xi) d\xi + kM \int_0^{q^*_{BPL}} (1 - \xi) d\xi \right] - n\alpha {q_e^*}^2. \tag{7}$$

To understand the role of a scheme like the GSP, we first study the market outcome in the absence of any intervention (i.e., the "free-market" outcome). We then demonstrate the reasons that motivate the need for a market intervention.

Remark 3.1. Recall, from the sequence of events, that we make two assumptions about the social planner:

[5] For ease of notation, we suppress the dependencies of the surplus on the equilibrium decisions that arise as a result of the social planner's decision.

(a) the social planner distributes *all* of the procured quantity among the BPL consumers, and

(b) the social planner distributes the procured quantity at *zero cost*.

Both these assumptions are not restrictive, in the sense that our conclusions qualitatively stay the same under a more-general setting where:

(a') the social planner commits to distributing a fraction, say $x_S \leq 1$, of the procured quantity among the BPL consumers, and

(b') charging a nominal positive price, say $p_S \geq 0$, to the BPL consumers.

We briefly outline how the analysis can be extended to the general setting. Let q'_{BPL} denote a BPL consumer's demand in the open market. Since p_S is sufficiently small, a BPL consumer's demand in the open-market is $q'_{BPL} = \min\left\{1 - q_S - p_m, \frac{b - p_S q_S}{p_m}\right\}$ such that the total quantity that the social planner distributes among the BPL consumers is $kMq_S \leq x_S n q_g$. Observe that the generalizations (a') and (b') above act in opposite directions in how they affect a BPL consumer's demand. All else equal, relative to the case where $x_S = 1$ and $p_S = 0$, a value of $x_S < 1$ results in the social planner distributing a lower quantity among the BPL consumers. Hence, the first term in the BPL consumer's demand in the above expression is higher, i.e., BPL consumers have a greater need for foodgrains in the open-market. On the other hand, a value of $p_S > 0$ results in lower wealth among the BPL consumers for purchase in the open-market. Hence, the second term in the BPL consumer's demand in the above expression is lower. Consequently, the equilibrium market outcome depends on the exact values of x_S and p_S and also whether the poorness of the BPL consumers is a dominant factor. ■

Remark 3.2. **(Connection to Price Floors)** A price floor for a good is an intervention through which the government imposes a lower bound on its market price. For it to be effective, a price floor should clearly be higher than the free-market equilibrium price. From standard microeconomic theory we know that, under such a price floor, the equilibrium production is higher while the demand is lower, relative to the case where the price floor is absent. Consequently, compared to the outcome in a free market, the consumer surplus is lower, the producer surplus is higher, and the total social welfare is lower; the loss in social welfare from a price floor is referred to as the "dead-weight" loss; see, e.g., Varian (1992).

Unlike in a price floor, the government does not regulate the market price directly under the GSP scheme. Indeed, there are two outlets for the producers to sell under a GSP scheme: the open market and the government. The government offers a fixed support price, while the open market price is determined by supply and demand. On their part, the producers (farmers) can sell partial amounts in both these outlets in equilibrium. The other important change in the action of a GSP scheme comes from the structure of the demand side: The consuming population consists of two segments, namely the BPL and APL consumers. The amount procured by the government under the scheme is used to increase the consumption of only one of these two segments (the BPL consumers). As far as the operational decisions are concerned, the presence of two consumer segments and two selling markets for the producers — of which one (the open market) is accessible to both consumer segments and the other (the government) is accessible only to one segment (the BPL consumers) — makes the equilibrium analysis of the producers' selling decisions and the social planner's choice of the support price quite challenging. ■

4. Absence of an Intervention: The Laissez-Faire Outcome

In the absence of any market intervention (henceforth "No Intervention", or NI), the farmers sell their entire produce in the open market. Each farmer decides his (expected) profit-maximizing effort q_e, given his belief about the market price $\hat{p}_m(\gamma)$, by solving the following problem:

$$\max_{q_e \geq 0} \pi_f(q_e) = \mathbb{E}[\hat{p}_m(\gamma) q_e \gamma] - \alpha q_e^2, \tag{8}$$

where the term inside the expectation is the farmer's belief about his revenue when the realized yield is γ. The farmer's profit maximizing effort q_e^*, given his belief $\hat{p}_m(\gamma)$, is

$$q_e^* = \frac{\mathbb{E}[\hat{p}_m(\gamma)\gamma]}{2\alpha}. \tag{9}$$

Let $\mathcal{Q}$ denote the total production by the farmers; $\mathcal{Q} = nq_e^* = n\left(\frac{\mathbb{E}[\hat{p}_m(\gamma)\gamma]}{2\alpha}\right)$. From (9), farmers exert greater effort if they believe that the market price is higher. Notice that the largest value of $\mathcal{Q}$ is $\frac{n}{2\alpha}$ (which occurs if γ is deterministically equal to 1 and $\hat{p}_m(1) = 1$). Recall that the maximum consumption by any consumer is capped at 1. To avoid settings that result in excess supply, we make the following assumption.

Assumption 4.1. **(Large Consumer Population)** *The maximum production from the farmers does not exceed the maximum consumption by the consumers, i.e.,* $\frac{n}{2\alpha} < M(1+k)$.

This is a reasonable assumption, since many developing countries have a large consumer population.

When the realized yield is γ, the total quantity available to be sold in the open market is $\mathcal{Q}\gamma$. In the absence of any scheme, the BPL consumers do not receive any support from the social planner, i.e., $q_S = 0$. The market price $p_m(\gamma)$ is obtained from the following:

$$D(p_m(\gamma)) = \mathcal{Q}\gamma, \text{ and} \tag{10}$$

$$\hat{p}_m(\gamma) = p_m(\gamma) \tag{11}$$

(10) states that, at the prevailing market-price, the total demand from the consuming population is equal to the total production from the farmers (i.e., the market clears). (11) states that farmers' belief is consistent with the outcome (i.e., farmers hold rational beliefs). Using these two conditions, we can obtain the equilibrium of the game, i.e., the equilibrium effort of the farmers and the equilibrium market price (consistent with the farmers' belief) for any realized yield γ.

Two factors play a key role in determining the equilibrium effort of the farmers – yield uncertainty, as a supply-side impediment, and the "poorness" of the BPL consumers, as a demand-side impediment. In what follows, we isolate the role of each of these factors by solving the equilibrium of this game in two special cases. We then explain the significance of the two results below (namely, Lemmas 4.1 and 4.2) that motivate the need for an intervention.

4.1. Effect of Yield Uncertainty

To isolate the effect of yield uncertainty on the equilibrium outcome, suppose that the wealth b of a BPL consumer is sufficiently high – in this case, the BPL consumers are effectively not budget-constrained.[6] Therefore, from (4), we have $q^*_{BPL} = 1 - p_m$. Let $\mu = \mathbb{E}[\gamma]$ and $\sigma^2 = \mathrm{Var}(\gamma)$. The following result states the equilibrium outcome under this special case.

LEMMA 4.1. *In the absence of any intervention, if $b \geq \frac{1}{4}$, the equilibrium effort of a farmer is*

$$q_e^* = \frac{\mu}{2\alpha}\Big(\frac{M(1+k)}{M(1+k) + \frac{n}{2\alpha}(\mu^2+\sigma^2)}\Big) \tag{12}$$

and the equilibrium market price when the realized yield is γ is

$$p_m(\gamma) = 1 - \gamma\Big(\frac{\frac{n}{2\alpha}\mu}{M(1+k) + \frac{n}{2\alpha}(\mu^2+\sigma^2)}\Big). \tag{13}$$

When the BPL consumers have sufficient wealth, the only impediment to high production by the farmers is the uncertainty in yield. All else equal, the equilibrium effort of a farmer q_e^* increases in the mean μ and decreases in the variance σ^2. Consequently, as yield becomes more variable, the equilibrium production effort by the farmers decreases – this argues for an incentive to improve production under yield uncertainty.

4.2. Effect of "Poorness" of the BPL Consumers

To better understand the effect of limited wealth of the BPL consumers and in the rest of the paper, we consider a two-point distribution for the yield.[7] Suppose that

$$\gamma = \begin{cases} 1, \text{ w.p. } \theta; \\ 0, \text{ w.p. } (1-\theta). \end{cases}$$

[6] A sufficient condition under which this occurs is $b \geq \frac{1}{4}$; under this condition, we have $\frac{b}{p_m} \geq (1-p_m)$, since $p_m \in [0,1]$.

[7] Some analysis under a general yield distribution is provided in Appendix C.

We define a threshold level of the budget, denoted by $b^*(\theta)$, as follows:

$$b^*(\theta) = \frac{M(1+k)\frac{n\theta}{2\alpha}}{\left(M(1+k)+\frac{n\theta}{2\alpha}\right)^2}. \tag{14}$$

Observe that $b^*(\theta)$ is a strictly increasing function of θ, $b^*(0)=0$ and $b^*(1)<\frac{1}{4}$, since $\frac{n}{2\alpha}<M(1+k)$. Intuitively, for a fixed $\theta\in[0,1]$, $b^*(\theta)$ denotes the maximum value of b for a BPL consumer to be deemed as "budget-constrained". In other words, if $b\geq b^*(\theta)$, then the BPL consumers are effectively not budget-constrained; therefore, at his equilibrium consumption q^*_{BPL}, a BPL consumer's marginal consumption utility equals the market price p_m.

From (9), we have that $q^*_e=\frac{\theta\hat{p}_m(1)}{2\alpha}$. The following result states the equilibrium outcome under this setting.

Lemma 4.2. *Consider a fixed value of $\theta\in[0,1]$. In the absence of any intervention, the equilibrium effort and the market price are as follows:*

1. *If $b<b^*(\theta)$, then*
$$q_e^{*NI}=\frac{\theta}{2\alpha}\left(\frac{M+\sqrt{M^2+4kMb(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}\right),$$
and the equilibrium market price (under the high yield realization) is
$$p_m(1)^{NI}=\frac{M+\sqrt{M^2+4kMb(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}. \tag{15}$$
2. *If $b\geq b^*(\theta)$, then*
$$q_e^{*NI}=\frac{\theta}{2\alpha}\left(\frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}\right),$$
and the equilibrium market price (under the high yield realization) is
$$p_m(1)^{NI}=\frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}. \tag{16}$$

Combining (15) and (16), we can succinctly state the outcome under NI as $\mathcal{Q}^{NI}=n\frac{\theta p_m(1)^{NI}}{2\alpha}$ where

$$p_m(1)^{NI}=\min\left\{\frac{M+\sqrt{M^2+4kMb(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})},\frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}\right\}. \tag{17}$$

Lemma 4.2 is interesting for several reasons:

(a) **BPL Consumers' Poverty as a Demand-Side Impediment to Production:** Suppose that $b<b^*(\theta)$: In this case, the BPL consumers are "budget-contrained". Precisely, the marginal utility from consumption (for a BPL consumer) is strictly higher than the market price, i.e., $1-\frac{b}{p_m(1)}>p_m(1)$. Stated differently, if the BPL consumers had more wealth, they would *choose* to purchase more from the open market at the prevailing market price. Farmers rationally anticipate the low buying power of the BPL consumers and therefore their equilibrium production effort is lower. At low values of b, the production by the farmers and the consumption by the BPL consumers is low, which justifies the need for an intervention by the social planner, both as a supply-side and a demand-side stimulant. As b increases, the buying power of the BPL consumers also increases. Consequently, the equilibrium production effort by the farmers also increases, i.e., q_e^* is increasing in b. Beyond $b^*(\theta)$, yield uncertainty plays a dominant role. As a result, beyond $b^*(\theta)$, the equilibrium effort q_e^* is a constant.

(b) **BPL Consumers' Poverty vis-à-vis Yield Uncertainty as Impediments to Production:** Consider a fixed $b \in [0, b^*(1)]$ and let $\theta^* = {b^*}^{-1}(b)$: The result states that BPL consumers are "budget-constrained" only when $\theta > \theta^*$. A consequence of this result is that if a high yield realization is less likely to occur (i.e., $\theta < \theta^*$), an increase in the wealth of the BPL consumers has no effect on the equilibrium outcome.

(c) **BPL Consumers' Poverty as an Externality to the APL Consumers' Consumption:** Consider a fixed value of θ and suppose that $b < b^*(\theta)$: A decrease in b leads to a decrease in the equilibrium effort q_e^* and a decrease in the market price $p_m(1)$. Consequently, the quantity consumed by an APL consumer, $1 - p_m(1)$, increases. In other words, as BPL consumers become poorer, the total production effort by the farmers decreases but the equilibrium consumption of the APL consumers increases.

To summarize, Lemmas 4.1 and 4.2 justify the need for an intervention in light of the yield uncertainty and the limited wealth of the poor consumers. In what follows, we analyze the Direct Benefit Transfer mechanism (DBT) as a benchmark intervention and then proceed with the analysis of the Guaranteed Support Price (GSP) scheme.

5. Benchmark Intervention: The Direct Benefit Transfer Scheme[8]

Consider the social planner fueled by a budget $B(> 0)$. Let $\beta = \frac{B}{kM}$. Under the Direct Benefit Transfer (DBT) scheme, the social planner augments each BPL consumer's wealth using his budget – therefore, the wealth of each BPL consumer becomes $b + \beta$.[9] As is the case in developing countries, we assume that the budget for the DBT scheme is limited – consequently, β is small relative to b. Specifically, we make the following assumption:

[8] We thank an anonymous reviewer for suggesting this comparison.

[9] An alternate cash transfer scheme is one where the social planner distributes his budget among the farmers, instead of the BPL consumers, as considered in this section. However, it should be obvious to the reader that, relative to NI, such a scheme does not alter incentives of any player. Therefore, the market outcome under such a scheme is identical to NI, except that the utility of each farmer increases by an amount equal to the wealth he receives from the social planner, i.e., B/n (under NI, the budget was left unused by the social planner). Consequently, the social planner's surplus is also identical to that under NI. Further, in Section 6, we show that this alternate scheme can be theoretically implemented by the GSP scheme as a special case.

Assumption 5.1. **(Limited Budget)** *The budget B of the social planner is less than the total value of trade that occurs in the open-market under NI. That is,*

$$B < \underbrace{p_m(1)^{NI}\, \mathcal{Q}^{NI}}_{\text{market price} \times \text{quantity of foodgrains}} \tag{18}$$

$$\textit{Using (17), we have:} \quad B < n\frac{\theta}{2\alpha}\left(\min\left\{\frac{M+\sqrt{M^2+4kMb(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}, \frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}\right\}\right)^2$$

From (15) and (16), $p_m(1)^{NI}\, \mathcal{Q}^{NI}$ (the RHS of (18)) is increasing in θ. Therefore, (18) can be equivalently stated as $\theta > \underline{\theta}$ for some $\underline{\theta} \in (0,1)$, i.e., the yield uncertainty is not acute.

Recall the definition of $b^*(\theta)$ from (14). For a fixed θ, one of the following occurs:

(a) $b < b+\beta < b^*(\theta)$,

(b) $b \le b^*(\theta) \le b+\beta$, and

(c) $b^*(\theta) < b < b+\beta$.

Since β is small relative to b, we ignore case (b) and focus on cases (a) and (c). Recall, from Lemma 4.2, that under NI, the effect of an increase in the wealth of the BPL consumers on the market outcome depends on the relative comparison between b and $b^*(\theta)$. Therefore, we analyze cases (a) and (c) separately.

5.1. Case (a): $b+\beta < b^*(\theta)$ (Poorness is Extreme)

Recall that under NI, this setting corresponds to the case where the "poorness" of the BPL consumers plays a role in determining the equilibrium effort of the farmers (case 1 in Lemma 4.2). Further, an increase in the BPL consumers' wealth results in an increase in the equilibrium production effort by the farmers (i.e., q_e^* is increasing in b). Therefore, it is straightforward to see that the DBT scheme – through which the social planner provides additional wealth to the BPL consumers – results in an increase in the production effort of the farmers.

Theorem 5.1 below compares the equilibrium outcomes and the social planner's surplus under DBT and NI. Under NI, the social planner's budget ($= kM\beta$) is left unused. Under DBT, the BPL consumers strictly prefer to purchase more from the open-market using the additional wealth β (instead of keeping all or part of it unused). Therefore, the surplus of BPL consumers under the DBT scheme exceeds the sum of their surplus under NI and the additional wealth β, i.e., ${u_{BPL}}^{DBT} > {u_{BPL}}^{NI} + \beta$. Relative to NI, the equilibrium effort of the farmers and the market-price are higher under DBT. Although their production costs are higher, the expected profit of the farmers increases due to larger revenues. However, the higher market-price results in the APL consumers being worse-off. Nevertheless, the total increase in the surplus of the BPL consumers and the farmers offsets the decrease in the surplus of the APL consumers and the unused budget; thus, the surplus of the social planner under the DBT scheme is strictly higher than that under NI.

THEOREM 5.1. *If $b+\beta < b^*(\theta)$, then the farmers' equilibrium effort under the DBT scheme is*

$$q_e^{*DBT} = \frac{\theta}{2\alpha}\left(\frac{M+\sqrt{M^2+4kM(b+\beta)(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}\right),$$

and the equilibrium market price (under the high-yield realization) is

$$p_m(1)^{DBT} = \frac{M+\sqrt{M^2+4kM(b+\beta)(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}. \tag{19}$$

Thus, relative to NI, the total production by the farmers and the market price (under the high-yield realization) are higher under the DBT scheme, i.e., $\mathcal{Q}^{DBT} > \mathcal{Q}^{NI}$ and $p_m(1)^{DBT} > p_m(1)^{NI}$. Further, the surplus of the social planner is strictly higher under the DBT scheme, i.e., $\Pi_S^{DBT} > \Pi_S^{NI}$.

5.2. Case (c): $b > b^*(\theta)$ (Yield Uncertainty is Dominant)

Under NI, this setting corresponds to the case where yield uncertainty is dominant in the determination of the equilibrium effort of the farmers and the BPL consumers are effectively not budget-constrained (case 2 in Lemma 4.2). Recall that, in this case, an increase in the BPL consumers' wealth has no effect on the market outcome. The additional wealth of the BPL consumers does not alter the incentives of the farmers to increase their production. Therefore, relative to NI, the DBT scheme does not alter the market outcome, i.e., there is no improvement in the production effort of the farmers or the social planner's surplus. The following result states this finding; the proof follows from Lemma 4.2.

THEOREM 5.2. *If $b > b^*(\theta)$, then the farmers' equilibrium effort under the DBT scheme and the market price under the high-yield realization are identical to the respective outcomes under NI. That is,*

$$q_e^{*DBT}(= q_e^{*NI}) = \frac{\theta}{2\alpha}\left(\frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}\right) \text{ and}$$

$$p_m(1)^{DBT}(= p_m(1)^{NI}) = \frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}} \tag{20}$$

Consequently, the total production by the farmers and the social planner's surplus under the DBT scheme is identical to that under NI, i.e., $\mathcal{Q}^{DBT} = \mathcal{Q}^{NI}$ and $\Pi_S^{DBT} = \Pi_S^{NI}$.

In summary, relative to NI, the DBT scheme leads to a strict improvement in the production effort of the farmers and the social planner's surplus if the poorness of the BPL consumers and yield uncertainty both play a role in determining the equilibrium effort of the farmers and the market price. However, if the poorness of the BPL consumers does not play a role (i.e., only yield uncertainty is a factor), then the DBT scheme is ineffective in improving the market outcome. Next, we study how the GSP scheme affects the market outcome in both these cases. We then contrast the outcome under the GSP scheme with that under the DBT scheme and under NI.

6. The Guaranteed Support Price Scheme: Analysis

Below, we analyze the market outcome under the GSP scheme for an arbitrary announced value of the support price p_g (i.e., both on- and off-equilibrium values of p_g). We use these results to identify the equilibrium value of p_g and compare the market outcomes under the GSP and DBT schemes.

Recall the sequence of events under the GSP scheme: The social planner announces a support price p_g ahead of the sowing season. Using this price, the farmers form rational beliefs about the market price – let $\hat{p}_m(1)$ (resp., $\hat{p}_m(0)$) denote the belief about the market price under the high-yield (resp., low-yield) realization. Nature chooses the realized yield γ, and the farmers make their selling decisions (q_g, q_m). The realized open-market price is consistent with the farmers' beliefs. It is straightforward that $\hat{p}_m(0) = 1$ and $\hat{p}_m(1) \leq 1$. For any choice p_g of the social planner, we have one of the following cases:

(*i*) $p_g < \hat{p}_m(1)$: The farmers do not sell to the social planner and sell their entire output in the open-market.

(*ii*) $p_g > \hat{p}_m(1)$: The farmers sell the maximum possible quantity to the social planner, and then sell any remaining quantity in the open market.

(*iii*) $p_g = \hat{p}_m(1)$: The farmers are indifferent between selling to the social planner and in the open-market.

In the result below, we solve for the farmer's effort and the selling decisions given his belief about the market price and the announced support price.[10]

Lemma 6.1. *For an announced p_g and beliefs $(\hat{p}_m(0), \hat{p}_m(1))$ about the market price, the farmer's equilibrium effort (solution to* **`Problem`** P_f^1*) and his selling decisions (solution to* **`Problem`** P_f^2*) are as follows:*

[10] As remarked in Section 3.2, an alternative is to consider asymmetric pure strategies, where we solve for the proportion of farmers who sell in the open market and to the social planner – such a consideration leads to an identical market outcome.

(i) *If* $p_g < \hat{p}_m(1)$, *then* $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$, $q_g = 0$, *and* $q_m = q_e$.

(ii) *If* $p_g > \hat{p}_m(1)$, *then*

$$q_e^* = \begin{cases} \frac{\theta}{2\alpha}p_g, & \frac{\theta}{2\alpha}\hat{p}_m(1) \le \frac{\theta}{2\alpha}p_g \le \frac{B}{np_g}; \\ \frac{B}{np_g}, & \frac{\theta}{2\alpha}\hat{p}_m(1) \le \frac{B}{np_g} \le \frac{\theta}{2\alpha}p_g; \\ \frac{\theta}{2\alpha}\hat{p}_m(1), & \frac{B}{np_g} \le \frac{\theta}{2\alpha}\hat{p}_m(1) \le \frac{\theta}{2\alpha}p_g. \end{cases}$$

$q_g = \min\left\{q_e, \frac{B}{np_g}\right\}$, *and* $q_m = q_e - q_g = \max\{0, q_e - \frac{B}{np_g}\}$.

(iii) *If* $p_g = \hat{p}_m(1)$, *then* $q_e^* = \frac{\theta}{2\alpha}p_g$ *and any choice of* (q_g, q_m) *such that* $q_g, q_m \ge 0$ *and* $q_g + q_m = q_e$ *is an equilibrium.*

Below, we solve and obtain the equilibrium market outcome for any choice of the support price announced by the social planner. Then, we find the equilibrium support price that maximizes the social planner's surplus. The analysis of the market outcome for both on- and off-equilibrium support prices is useful because Section 7 considers an alternate objective function of the social planner (a weighted combination of the BPL consumers' surplus and the farmers' surplus).

6.1. Case (a): $b + \beta < b^*(\theta)$ (Poorness is Extreme)

From Lemma 4.2 and Theorem 5.1, recall that if the BPL consumers' poorness plays a dominant role in determining the equilibrium effort of the farmers, then the DBT scheme leads to a strict improvement in the production by the farmers and the social planner's surplus relative to NI. Further, recall the expressions for the market prices under NI and DBT (i.e., $p_m(1)^{NI}$ and $p_m(1)^{DBT}$) from (15) and (19).

In the result below, we obtain the market outcome and the social planner's surplus under any value of the support price announced by the social planner. Subsequently, we use these results to determine the equilibrium support price.

Lemma 6.2. *If* $b + \beta < b^*(\theta)$, *then for any announced support price* p_g, *the market outcome in the corresponding subgame is as follows:*

1. **Production and Selling Decisions:**
   (i) *If* $p_g < p_m(1)^{NI}$, *then* $q_e^* = \frac{\theta}{2\alpha}p_m(1)^{NI}$, $q_g^* = 0$, $q_m^* = q_e^*$.
   (ii) *If* $p_g \in \left[p_m(1)^{NI}, p_m(1)^{DBT}\right]$, *then* $q_e^* = \frac{\theta}{2\alpha}p_g$, $q_g^* = \frac{\theta}{2\alpha}p_g - \frac{M}{n}(1-p_g) - \frac{kMb}{np_g}$, $q_m^* = q_e^* - q_g^*$.
   (iii) *If* $p_g > p_m(1)^{DBT}$, *then* $q_e^* = \frac{\theta}{2\alpha}p_m(1)$, $q_g^* = \frac{B}{np_g}$, $q_m^* = q_e^* - q_g^*$, *where*

$$p_m(1) = \frac{\left(M + kM\frac{\beta}{p_g}\right) + \sqrt{\left(M + kM\frac{\beta}{p_g}\right)^2 + 4kMb\left(M + \frac{n\theta}{2\alpha}\right)}}{2\left(M + \frac{n\theta}{2\alpha}\right)} \tag{21}$$

2. **Social Planner's Surplus:**

(i) *If* $p_g < p_m(1)^{NI}$*, then* ${\Pi_S}^{GSP}(p_g) = {\Pi_S}^{NI}$*.*

(ii) *If* $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$*, then* ${\Pi_S}^{GSP}(p_g)$ *increases in* p_g*, from* ${\Pi_S}^{NI}$ *at* $p_g = p_m(1)^{NI}$ *to* ${\Pi_S}^{DBT}$ *at* $p_g = p_m(1)^{DBT}$*.*

(iii) *If* $p_g > p_m(1)^{DBT}$*, then* ${\Pi_S}^{GSP}(p_g)$ *decreases in* p_g*, from* ${\Pi_S}^{DBT}$ *at* $p_g = p_m(1)^{DBT}$ *and approaches* ${\Pi_S}^{NI}$ *as* $p_g \to \infty$*.*

Some comments are in order.

(i) If the support price p_g is below $p_m(1)^{NI}$ (the market price under NI), then the farmers anticipate a higher market price, and therefore do not sell to the social planner. Consequently, the entire budget of the social planner is left unused. The market outcome, and therefore the social planner's surplus, under such low values of p_g, are identical to that under NI.

(ii) If $p_g \in \left[p_m(1)^{NI}, p_m(1)^{DBT}\right]$, then relative to NI, the social planner distorts the market outcome under GSP. The production effort by the farmers increases (linearly) in p_g. However, the quantity they sell in the open market decreases. Therefore, the market price is increasing in p_g. If $p_g = p_m(1)^{NI}$, the market outcome is identical to that under NI where the social planner's budget is unused; thus, ${\Pi_S}^{GSP} = {\Pi_S}^{NI}$. If $p_g = p_m(1)^{DBT}$, then the market outcome is identical to that under DBT, where the social planner exhausts his budget. Therefore, $\Pi_S^{GSP} = {\Pi_S}^{DBT}$. An intermediate value of p_g leads to an interesting comparison: Consider any $p_g \in \left[p_m(1)^{NI}, p_m(1)^{DBT}\right]$ and define $\delta(p_g)$ as follows:

$$\delta(p_g) = \frac{-b}{\beta} + \frac{1}{kM\beta}\left(\frac{n\theta}{2\alpha}p_g^2 - M(1-p_g)p_g\right).$$

It is straightforward to verify that $\delta(p_g)$ is increasing in p_g, $\delta(p_m(1)^{NI}) = 0$, and $\delta(p_m(1)^{DBT}) = 1$. At such an intermediate value of p_g, the social planner uses an amount $\delta(p_g)B$ in procuring from the farmers, while $(1-\delta(p_g))B$ is left unused. Consequently, the social planner's surplus under the GSP scheme is increasing in p_g.

(iii) Finally, as p_g increases beyond $p_m(1)^{DBT}$, the social planner's budget is fully exhausted. Hence, the quantity procured by the social planner $\frac{B}{p_g}$ decreases in p_g. Consequently, the production effort of the farmers also decreases in p_g. The quantity sold in the open market increases and hence, the market price decreases in p_g. In the limit (as $p_g \to \infty$), the social planner procures very little from the farmers, and hence the outcome is identical to the outcome under an alternate distribution scheme (see footnote 9) where the social planner distributes the entire budget among the farmers. The market outcome in the limit (as $p_g \to \infty$) is identical to that under NI, except that the social planner's budget (which is unused under NI) is distributed among the farmers.

Based on the discussion above, the following result states the equilibrium value of p_g.

Theorem 6.1. **(Equilibrium Support Price)** *If $b+\beta<b^*(\theta)$, then the equilibrium support price $p_g=p_m(1)^{DBT}$. Further, the equilibrium production by the farmers and the social planner's surplus is identical to that under DBT and strictly higher than that under NI, i.e., $\mathcal{Q}^{GSP}=\mathcal{Q}^{DBT}>\mathcal{Q}^{NI}$ and $\Pi_S^{GSP}=\Pi_S^{DBT}>\Pi_S^{NI}$.*

6.2. Case (c): $b>b^*(\theta)$ (Yield Uncertainty is Dominant)

Recall from Section 5.2 that if $b>b^*(\theta)$, then the DBT scheme is ineffective in improving the market outcome relative to NI. Specifically, the production effort of the farmers, the market price, and the social planner's surplus under the DBT scheme are all identical to those under NI. Further, recall the expression for the market price under NI (identical to that under DBT) from (16). In what follows, we analyze the outcome under the GSP scheme. Lemma 6.3 below identifies the market outcome under any announced value of the support price p_g. This result helps us in obtaining the equilibrium value of the support price (Theorem 6.2).

Lemma 6.3. *If $b>b^*(\theta)$ and Assumption 5.1 holds, the market outcome for any $p_g\geq 0$ is as follows:*

- **Production and Selling Decisions:** *The equilibrium effort of the farmers is $q_e^*=\frac{\theta}{2\alpha}p_m(1)^{NI}$, and the market price is $p_m(1)=p_m(1)^{NI}$.*
- **Social Planner's Surplus:** $\Pi_S^{GSP}(p_g)=\Pi_S^{NI}$.

We explain this result below.

(i) If $p_g<p_m(1)^{NI}$, it is straightforward that the farmers anticipate a higher market price and therefore do not sell to the social planner. The entire budget of the social planner is unused and, hence, the outcome is identical to that under NI.

(ii) If $p_g\geq p_m(1)^{NI}$, then the farmers anticipate a downward shift in the BPL consumers' demand caused by the additional quantity they receive from the social planner. This downward shift is equal to the quantity that the farmers sell to the social planner. Therefore, relative to NI, the equilibrium effort of the farmers does not change. Consequently, the market outcome is also identical to that under NI.

Although the social planner's surplus for any support price under GSP is identical to that under NI, the BPL consumers' surplus and farmers' profits are strictly higher. To see this, consider a fixed $p_g>p_m(1)^{NI}$.

BPL Consumers' Surplus: Although the consumption quantity are identical to that under NI, the BPL consumers purchase a *smaller* quantity in the open-market. Thus, relative to NI, the

BPL consumers' surplus under the GSP scheme is higher. Under the high-yield realization, the quantity received by a BPL consumer from the social planner and the quantity he purchases from the open-market (from (4)) are:

$$q_S = \frac{\beta}{p_g} \text{ and } q^*_{BPL} = 1 - \frac{\beta}{p_g} - p_m(1)^{NI}.$$

Therefore, an individual BPL consumer's surplus can be written as

$$\begin{aligned} u_{BPL}(q^*_{BPL}) &= \theta\left(u_C\left(q_S + q^*_{BPL}\right) + b - p_m(1)^{NI} q^*_{BPL}\right) + (1-\theta)b \\ &= u^{NI}_{BPL} + \theta\left(\frac{p_m(1)^{NI}}{p_g}\right)\beta \end{aligned} \tag{22}$$

A BPL consumer is, therefore, "richer" under the GSP scheme (relative to NI) by an amount $\theta\left(\frac{p_m(1)^{NI}}{p_g}\right)\beta$.

Farmers' Profit: The cost of production for each farmer is identical to that under NI (since the production effort is identical) but they sell a fraction of their produce to the social planner at the support price (which is higher than the market price). That is, the equilibrium profit of a farmer can be written as

$$\begin{aligned} \pi^{GSP}_f &= -\alpha\left(\frac{\theta p_m(1)^{NI}}{2\alpha}\right)^2 + \theta\left(p_m(1)^{NI} \underbrace{\left(\frac{\theta p_m(1)^{NI}}{2\alpha} - \frac{B}{np_g}\right)}_{=q_m} + p_g \underbrace{\left(\frac{B}{np_g}\right)}_{=q_g}\right) \\ &= \pi^{NI}_f + \theta\frac{B}{np_g}\left(p_g - p_m(1)^{NI}\right) \end{aligned} \tag{23}$$

Indeed, the social planner can use the GSP scheme as a mechanism to divide his budget in any desired proportion to improve the surplus of the BPL consumers and the farmers; the exact split is achieved by setting an appropriate value of the support price. Precisely, let $\zeta \in [0,1]$ define this split: That is, $\zeta\theta B$ (resp., $(1-\zeta)\theta B$) denote the improvement in the surplus of the BPL consumers (resp., farmers) that the social planner intends to achieve relative to NI. The following result identifies the support price that achieves this split.

THEOREM 6.2. **(Equilibrium Support Price)** *If $b > b^*(\theta)$ and Assumption 5.1 holds, then there are a continuum of social planner's surplus-equivalent equilibria, where the social planner's surplus is equal to Π^{NI}_S. These equilibria differ in the relative benefit they provide to the BPL consumers and the farmers. While under NI, the social planner's budget is unspent, under GSP, the social planner can divide his budget in any proportion $\zeta \in [0,1]$ to achieve corresponding improvements in the surplus of the BPL consumers and the farmers, by choosing $p_g = \frac{p_m(1)^{NI}}{\zeta}$.*

On the one end, if the social planner intends to allot the entire budget to improve the utility of the BPL consumers (i.e., $\zeta = 1$), then $p_g = p_m(1)^{NI}$. On the other end, if the social planner only seeks to improve the utility of the farmers (i.e., $\zeta = 0$), then $p_g \to \infty$.

6.3. A General Two-Point Distribution

In Theorems 6.1 and 6.2, we show that the equilibrium production quantity and the social planner's surplus are identical under the DBT and the GSP schemes. Our analysis thus far assumes a $\{0,1\}$ (i.e., Bernoulli) yield distribution. In this section, we consider a more general two-point distribution for the yield γ with support $\{\gamma_L, \gamma_H\}$ as follows:

$$\gamma = \begin{cases} \gamma_L, & \text{w.p. } 1-\theta; \\ \gamma_H, & \text{w.p. } \theta. \end{cases}$$

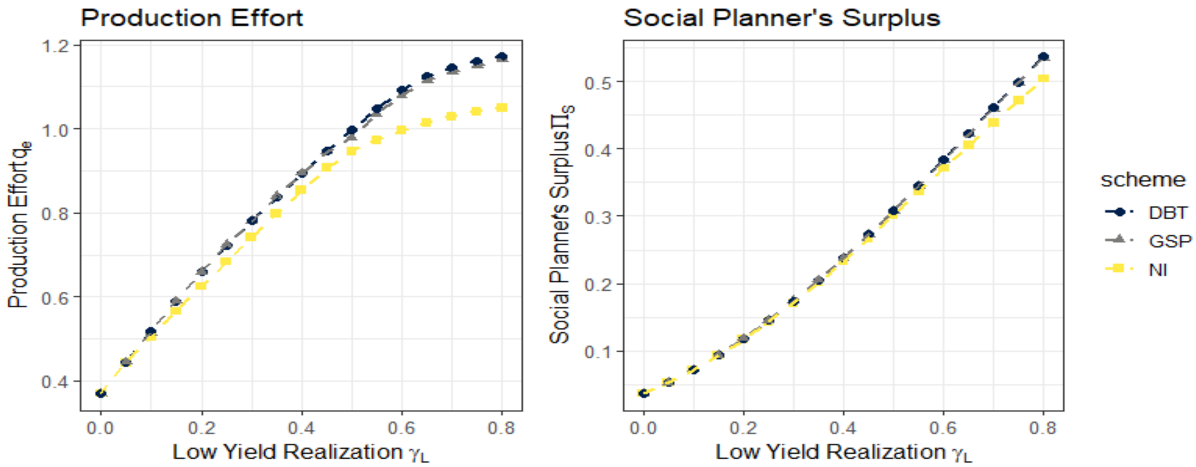
Using numerical experiments, we compare the production effort q_e and the social planner's surplus Π_S under the three settings, namely NI, DBT and GSP. We fix $\gamma_H (= 1)$ and numerically compare q_e and Π_S under each setting for $(\gamma_L, \theta) \in \{0, 0.05, 0.1, \ldots, 0.85\} \times \{0.2, 0.4, 0.6\}$; see Figure 1. Under NI and DBT, we endow the farmers with arbitrary (initial) beliefs on the market prices (i.e., assign arbitrary beliefs to $\hat{p}_m(\gamma)$ for $\gamma \in \{\gamma_L, \gamma_H\}$). Farmers' choose their production effort q_e to maximize their expected profit. We solve for the rational (equilibrium) beliefs on the market prices such that the farmers' effort leads to market prices that are consistent with their beliefs (i.e., for $\gamma \in \{\gamma_L, \gamma_H\}$, (10) and (11) hold). Under GSP, we solve for the rational beliefs corresponding to any support price p_g (i.e., on- or off-equilibrium). We detail the simulation procedure in Appendix D.

It is straightforward that q_e and Π_S are increasing in γ_L for a given θ and increasing in θ for a given γ_L under NI, DBT and GSP. Further, based on our numerical experiments, we find that the difference $q_e^{GSP} - q_e^{NI}$ (resp., $q_e^{DBT} - q_e^{NI}$) and $\Pi_S^{GSP} - \Pi_S^{NI}$ (resp., $\Pi_S^{DBT} - \Pi_S^{NI}$) is increasing with γ_L for given θ and is increasing in θ for a given γ_L. Our main insights based on the Bernoulli yield distribution in Theorems 6.1 and 6.2 – the equivalence of the GSP and the DBT schemes in terms of the total production by the farmers and the social planner's surplus – are robust to the more general yield distribution.

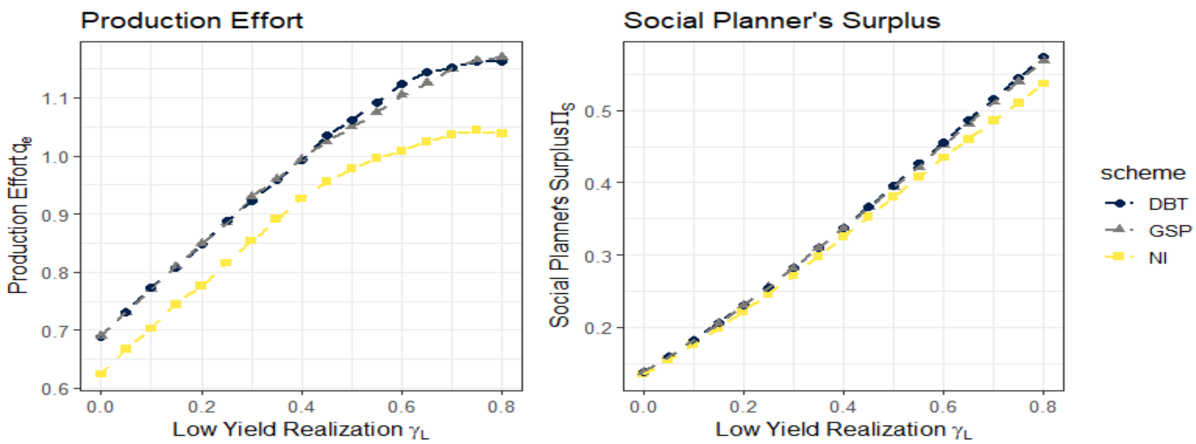
7. A Weighted Objective of the Social Planner[11]

In our analysis thus far, we have focused on a social planner whose objective consists of four components: the APL and BPL consumers' surplus, the farmers' surplus, and the unused budget. Recall from Section 1 that the GSP scheme is primarily intended to benefit two groups: the BPL consumers and the farmers. Therefore, we consider an alternate objective function of the social planner where he weighs the surplus of each segment differently. Specifically, we discuss a special

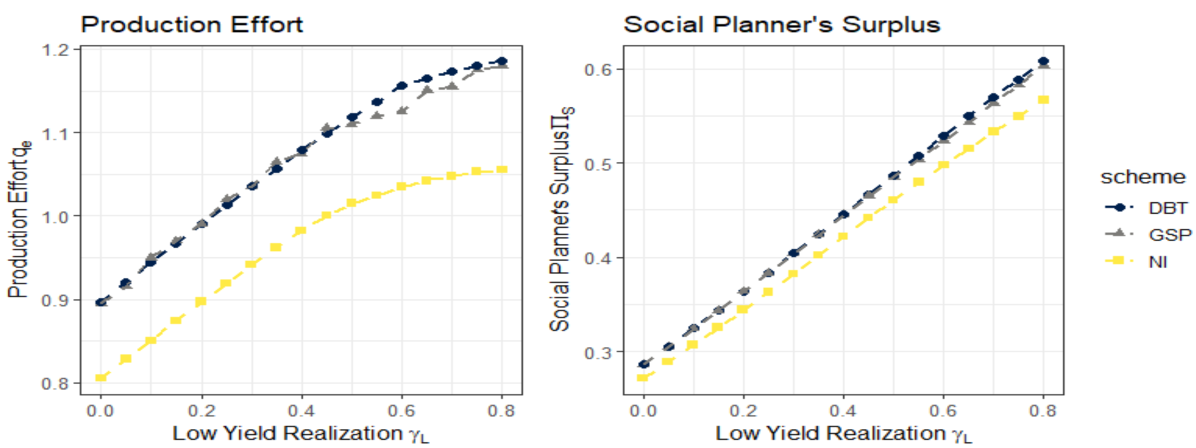
[11] We thank the review team for suggesting us to consider a weighted objective function for the social planner.



(a) $\theta = 0.2$.



(b) $\theta = 0.4$.



(c) $\theta = 0.6$.

Figure 1 **Comparison of the Production Effort (q_e) and Social Planner's Surplus (Π_S) under NI, DBT and GSP with γ_L. The parameters used are:** $M = 1$, $k = 4$, $b = 0.08$, $\beta = 0.04$, $\alpha = 0.25$, $n = 1$, $\gamma_H = 1$.

case where the social planner weighs the surplus of the farmers by a factor $\omega_F \in [0,1]$, the surplus of the BPL consumers by $(1-\omega_F)$, and the other components by 0.[12]

Then, for any $\omega_F \in [0,1]$, the social planner's problem is:

$$\max_{p_g \geq 0} \; \Pi_S(\omega_F) = \mathbb{E}_\gamma \left[\omega_F(n\pi_f) + (1-\omega_F)(kMu_{BPL}) \right]. \tag{24}$$

7.1. Case (a): $b+\beta < b^*(\theta)$ (Poorness is Extreme)

From Lemma 6.2, recall the following: The farmers' surplus is strictly increasing in p_g if $p_g \geq p_m(1)^{NI}$, while the BPL consumer surplus is increasing in p_g if $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$ and decreasing thereafter. Therefore, if $\omega_F = 0$ (resp., $\omega_F = 1$), then the equilibrium $p_g = p_m(1)^{DBT}$ (resp., $p_g \to \infty$). At an intermediate $\omega_F \in (0,1)$, the equilibrium support price p_g lies in the interval $\Big[p_m(1)^{DBT}, \infty\Big)$.

We illustrate this weighted objective function of the social planner as a function of ω_F and p_g in Figure 2 below. In our numerical experiments, we find that under sufficiently low (resp., sufficiently high) values of ω_F, the weighted social planner's surplus is strictly decreasing (resp., increasing) in p_g. Under intermediate values of ω_F, the social planner manages a tradeoff between the surplus of the BPL consumers (decreasing in p_g) with the surplus of the farmers (increasing in p_g). Consequently, the social planner's surplus corresponding to an intermediate value of ω_F is non-monotone in p_g (see the right panel of Figure 2).

7.2. Case (c): $b > b^*(\theta)$ (Yield Uncertainty is Dominant)

From the discussion preceding Theorem 6.2, recall that the GSP scheme enables the social planner to dedicate any desirable fraction $\zeta \in [0,1]$ of his budget for improving the surplus of the BPL consumers (and the remainder for improving the surplus of the farmers) by choosing $p_g = \left(\frac{p_m(1)^{NI}}{\zeta}\right)$. Consider any $p_g \geq p_m(1)^{NI}$; let $\zeta = \frac{p_m(1)^{NI}}{p_g}$. Using (22) and (23), we rewrite (24) in terms of ζ as follows:

$$\begin{aligned}
\Pi_S &= \omega_F \left(n{\pi_F}^{NI} + (1-\zeta)\theta B\right) + (1-\omega_F)\left(kM{u_{BPL}}^{NI} + \zeta\theta B\right) \\
&= \omega_F \left(n{\pi_F}^{NI}\right) + (1-\omega_F)kM{u_{BPL}}^{NI} + \theta\left(\omega_F B + 2B\zeta\left(\frac{1}{2} - \omega_F\right)\right)
\end{aligned}$$

[12] This choice of the social planner's objective allows us to study the effect of larger weights assigned to the surplus of the intended beneficiaries of the scheme on the equilibrium choice of the support price in a straightforward manner. Other choices of a weighted objective function, where the social planner assigns small, non-zero weights to the APL consumers' surplus and the unused budget involve more computation but yield qualitatively similar insights.

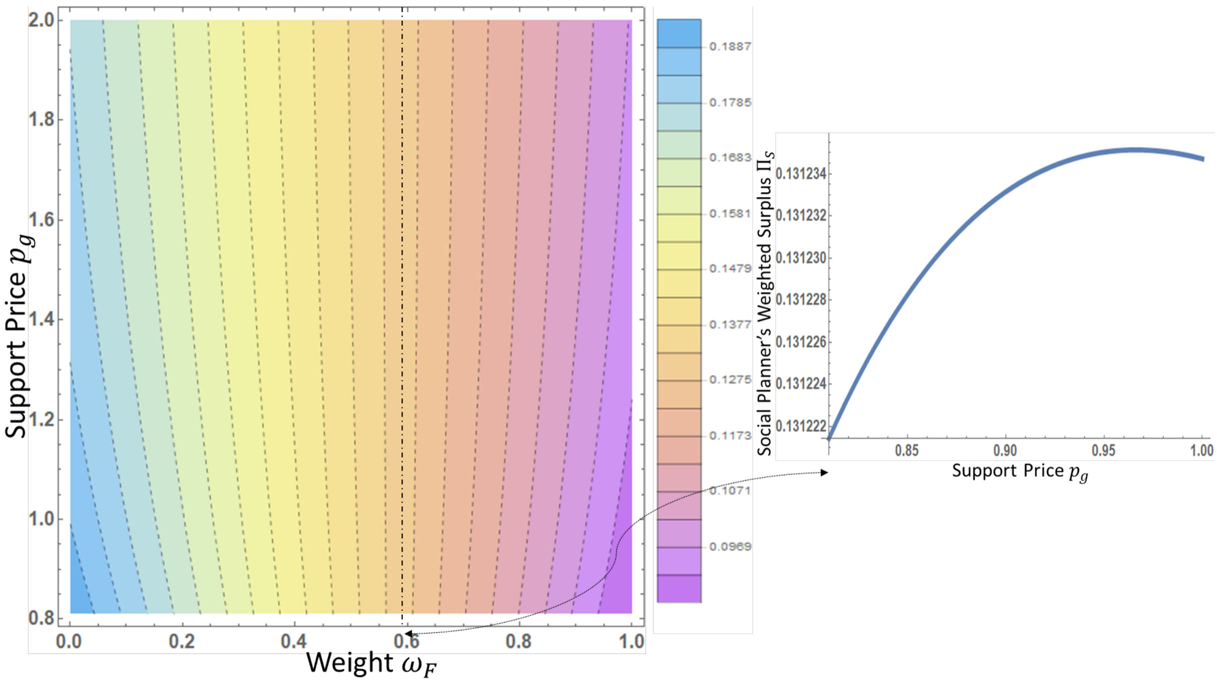


Figure 2 **Left: Weighted objective of the social planner as a function of the weight ω_F and the support price p_g. The numerical values of the various parameters are $B = 0.05$, $b = 0.03$, $k = 5$, $M = 1$, $n = 1$, $\alpha = 0.5$, $\theta = 0.5$. Under these numerical values, we have that $b + \beta = 0.04 < b^*(\theta) = 0.07$. Further, $p_m(1)^{DBT} = 0.81$, $p_m(1)^{NI} = 0.78$. Right: Weighted objective of the social planner at $\omega_F = 0.575$ as a function of the support price p_g.**

Consider a fixed $\omega_F \in [0,1]$: From above, it suffices to maximize $\zeta(\frac{1}{2} - \omega_F)$. Therefore, if $\omega_F \leq \frac{1}{2}$ (resp., $\omega_F > \frac{1}{2}$), then the social planner chooses $\zeta = 1$ (resp., $\zeta = 0$[13]). We state the equilibrium support price in the following result.

Lemma 7.1. *If $b > b^*(\theta)$, then the equilibrium support price under the weighted objective in (24) is as follows:*

$$p_g = \begin{cases} p_m(1)^{NI}, & \text{if } \omega_F \leq \frac{1}{2}; \\ \infty, & \text{if } \omega_F > \frac{1}{2}. \end{cases}$$

8. Real-World Considerations

Our analysis in Sections 5 and 6 suggests that the equilibrium surplus of the social planner under the DBT and the GSP schemes is identical. Below, we present two real-world situations that suggest the merits of the GSP scheme over the DBT scheme.

8.1. Reserve (or Buffer) Stock

Recall the twin goals of the GSP scheme: as a supply-side incentive, to ensure high output from the farmers, and a demand-side provisioning tool, to subsidize the consumption needs of the poor.

[13] $\zeta = 0$ (or equivalently, $p_g = \infty$) refers to the outcome where the social planner distributes the entire budget among the farmers.

Beyond these unexceptionable goals, it is also important for governments in these countries to maintain an adequate amount of foodgrains as reserve stock to mitigate the adverse effects due to yield uncertainty. According to Rakshit (2003):

> *"Food security – ironing out large intra- and inter-year variations in food prices through buffer stocking – encompasses a number of goals. One of these goals is of course consumption and price smoothing over a typical crop cycle."*.

It is important to note that the GSP scheme allows for the social planner to hold a buffer stock of foodgrains, while it is impossible to do so under the DBT scheme. Since our model captures the interactions during one crop cycle (i.e., a one-period model), we abstract away from the dynamics that occur inter-year and assume that the social planner derives an additional utility by maintaining a reserve stock of grains: Let I denote the quantity of foodgrains held as reserve stock by the social planner and $f(I)$ denote an additional surplus enjoyed by the social planner. That is, the social planner's surplus is

$$\max_{p_g,I} \Pi_S = \mathbb{E}_\gamma \left[M u_{APL} + kM u_{BPL} + n\pi_f + (B - np_g q_g) + f(I) \right]. \tag{25}$$

We assume that the function $f : [0,\infty) \to [0,\infty)$ has the following properties: $f(\cdot)$ is continuous and differentiable throughout, $f(0) = 0$, and $f(I)$ is (weakly) increasing in I. The analysis thus far corresponds to the case of $f(I) = 0$, i.e., the social planner does not gain any additional surplus due to a reserve stock of foodgrains. Therefore, the equilibrium value of I equals 0 and, consequently, the social planner's surplus under the equilibrium outcome of the GSP scheme is identical to that under the DBT scheme.

Suppose the social planner procures a quantity q_g from each farmer. Let δ_r denote the fraction of procured quantity withheld by the social planner as reserve stock; that is, $I = nq_g\delta_r$. If the realized yield is γ, the market clearance condition can be written as:

$$M(1 - p_m(\gamma)) + kM \min\left\{ 1 - p_m(\gamma) - \frac{n}{kM} q_g (1-\delta_r), \frac{b}{p_m(\gamma)} \right\} = n\left(\frac{\mathbb{E}[p_m(\gamma)\gamma]}{2\alpha}\gamma - q_g \right). \tag{26}$$

Theorem 8.1 below shows that if the value from holding a reserve stock of grains is sufficiently large, then the social planner will hold a positive amount of reserve stock; consequently, the GSP scheme outperforms the DBT scheme. Further, if yield uncertainty is dominant, then the equilibrium production by the farmers under GSP is also higher than that under DBT. Let

$$\underline{f} = \begin{cases} \frac{M + \frac{n\theta}{2\alpha}}{kM} \left(\frac{M(1+k)}{M + \frac{n\theta}{2\alpha}} - p_m(1)^{NI} \right), & \text{if } b + \beta < b^*(\theta); \\ \frac{M(1+k) - \frac{B}{p_m(1)^{NI}}}{M(1+k) + \frac{n\theta}{2\alpha}}, & \text{if } b > b^*(\theta). \end{cases}$$

Then, we have

Theorem 8.1. *If $f'(0) > \underline{f}$, then the equilibrium value of the reserve stock I under the GSP scheme is strictly positive. Therefore, $\Pi_S^{GSP} > \Pi_S^{DBT}$. Further, if $b > b^*(\theta)$, then $\mathcal{Q}^{GSP} > \mathcal{Q}^{DBT}$.*

8.2. Leakages in the DBT and the GSP Schemes

Our analysis considered a leakage-free environment under the DBT and the GSP scheme. However, in reality, leakages afflict both the schemes. We discuss them below.

- **Wealth Leakage in the DBT Scheme:** Muralidharan et al. (2017) show that beneficiaries incur a significant time and expense in order to access the cash subsidy of the DBT scheme (Finding 7 of their report). Taking these into account, they find that the DBT amount may be inadequate (relative to the equivalent grain through the GSP). Recipients of the cash subsidy also report inconsistency or low entitlements relative to expectations (Finding 3 and 6). Finally, recipients use/abuse some part of their cash subsidy on non-foodgrain purchases (socially non-beneficial activities such as gambling and alcohol).
- **Grain Pilferage in the GSP Scheme:** Several studies in economics have reported the extent and cost of pilferage in public distribution under the GSP scheme. For instance, Nagavarapu and Sekhri (2014) find that, in some regions, ration-shop holders and other participants in the distribution system illegally sell grain (meant for the BPL population) on the black market for high prices, resulting in a significant fraction of grain being diverted from the intended recipients. As an example, they estimate that for every Rs. 3.65 spent by the Government of India, only Rs. 1 reaches a BPL household. Distribution losses due to poor storage and transportation exacerbate such pilferages.

We model these leakages and show their effect on the performance of the respective schemes. For brevity, we consider the case where $b+\beta<b^*(\theta)$.

8.2.1. Wealth Leakage in the DBT Scheme Let $(1-\delta_\beta)$ (<1) denote the proportion of wealth that is actually spent on purchasing grains in the open market. The market clearance condition at $\gamma=1$ is as follows:

$$M(1-p_m(1))+kM\left(\frac{b+\beta(1-\delta_\beta)}{p_m(1)}\right)=n\theta\frac{p_m(1)}{2\alpha}. \tag{27}$$

The market price $p_m(1)^{DBT}\Big|_{\delta_\beta}$ (i.e., the solution to (27)) is identical to (19), except that $b+\beta$ is replaced by $b+\beta(1-\delta_\beta)$. Consequently, the total production quantity and the social planner's surplus is identical to that in Theorem 5.1, with $b+\beta$ replaced by $b+\beta(1-\delta_\beta)$. From (27), it is straightforward that $p_m(1)^{DBT}\Big|_{\delta_\beta}$ is decreasing in δ_β. Using (7), the social planner's surplus can be written as:

$$\Pi_S^{DBT}\Big|_{\delta_\beta}=(Mw_{APL}+kMb+B)-n\alpha\left(\frac{\theta p_m(1)}{2\alpha}\right)^2+\theta\left(M\int_0^{1-p_m(1)}(1-\xi)d\xi+kM\int_0^{\frac{b+\beta(1-\delta_\beta)}{p_m(1)}}(1-\xi)d\xi\right), \tag{28}$$

where $p_m(1) = p_m(1)^{DBT}\Big|_{\delta_\beta}$. Since $p_m(1)^{DBT}\Big|_{\delta_\beta}$ is decreasing in δ_β, the production cost (of the farmers) is decreasing in δ_β, the APL consumer surplus is increasing in δ_β, while the social planner's surplus $\Pi_S^{DBT}\Big|_{\delta_\beta}$ is decreasing in δ_β.

8.2.2. Grain Pilferage in the GSP Scheme Suppose that a fraction $(1-\delta_g)$ (<1) of the quantity procured by the social planner is distributed among the BPL consumers and the remainder is lost due to pilferage. Recall from Lemma 6.1 that in the absence of grain pilferage ($\delta_g = 0$) and when $b+\beta < b^*(\theta)$, the equilibrium support price $p_g = p_m(1)^{DBT}\Big|_{\delta_\beta=0}$ (shown in (19)). If $\delta_g > 0$, we have the following result about the equilibrium support price.

LEMMA 8.1. *In the presence of grain pilferage, there exists $\underline{\delta}_g \in (0,1)$ s.t. the equilibrium support price $p_g = p_m(1)^{DBT}\Big|_{\delta_\beta=0}$ if $\delta_g < \underline{\delta}_g$.*

Suppose $\delta_g < \underline{\delta}_g$. The social planner procures $\frac{B}{p_g}$; however, due to grain pilferage, the quantity received by a BPL consumer from the social planner is $q_S = \frac{\beta}{p_g}(1-\delta_g)$. Since $b+\beta < b^*(\theta)$, the BPL consumers spend their entire wealth in purchasing from the open-market. The market clearance condition at $\gamma = 1$ is:

$$M\left(1 - p_m(1)\right) + kM\frac{b}{p_m(1)} = n\frac{\theta p_m(1)}{2\alpha} - \frac{B}{p_g}. \tag{29}$$

Therefore, for any $\delta_g \in (0, \underline{\delta}_g)$, we have that $p_m(1)^{GSP}\Big|_{\delta_g} = p_g = p_m(1)^{DBT}\Big|_{\delta_\beta=0}$.

8.2.3. Comparison of the Schemes in the Presence of Leakages Comparing (27) and (29), we have that

$$p_m(1)^{DBT}\Big|_{\delta_\beta} < p_m(1)^{GSP}\Big|_{\delta_g}. \tag{30}$$

The following result compares the total production by the farmers under the two schemes in the presence of leakages.

THEOREM 8.2. **(Total Production Quantity)** *Suppose $b+\beta < b^*(\theta)$. Consider $\delta_g \in (0, \underline{\delta}_g)$ and $\delta_\beta \in (0,1)$. In the presence of leakages δ_g and δ_β, respectively, in the GSP and DBT schemes, the total production under the GSP scheme is strictly higher than that under the DBT scheme, i.e., $\mathcal{Q}^{GSP}(\delta_g) > \mathcal{Q}^{DBT}(\delta_\beta)$.*

The intuition behind Theorem 8.2 is as follows: Under the assumed conditions, the leakage δ_g in the GSP scheme leads to a decrease in the consumption by the BPL consumers, but does not affect the market price. That is, $p_m(1)^{GSP}\Big|_{\delta_g}$ is independent of δ_g. Therefore, the entire budget of the social planner and the wealth of the BPL consumers is spent in procuring grains from the farmers. On the other hand, the leakage in the DBT scheme leads to a decrease in the total wealth spent by the social planner and the BPL consumers, thereby affecting the market price. The farmers anticipate a lower market price, and hence lower their production.

The social planner's surplus under GSP can be written as:

$$\Pi_S^{GSP}\Big|_{\delta_g} = (Mw_{APL} + kMb + B) - n\alpha\left(\frac{\theta p_m(1)}{2\alpha}\right)^2 + \theta\left(M\int_0^{1-p_m(1)}(1-\xi)d\xi + kM\int_0^{\frac{b}{p_m(1)}+\frac{\beta}{p_g}(1-\delta_g)}(1-\xi)d\xi\right), \tag{31}$$

where $p_m(1) = p_m(1)^{GSP}\Big|_{\delta_g}$. Since $p_m(1)^{GSP}\Big|_{\delta_g}$ is independent of δ_g, the production cost of the farmers and the APL consumer surplus is independent of δ_g, while the BPL consumer surplus is decreasing in δ_g. Therefore, $\Pi_S^{GSP}\Big|_{\delta_g}$ is decreasing in δ_g.

We now compare the social planner's surplus under DBT and GSP in the presence of leakages in (28) and (31). The production cost under GSP is higher (from Theorem 8.2) and the APL consumer surplus under GSP is lower (from (30)). Therefore, for the social planner's surplus under the GSP to be higher than that under DBT, we require δ_β to be sufficiently high (rel. to δ_g).

Theorem 8.3. **(Social Planner's Surplus)** *Suppose* $b+\beta < b^*(\theta)$. *Consider any* $\delta_g \in (0, \underline{\delta}_g)$. *If* $\Pi_S^{GSP}\Big|_{\delta_g} > \Pi_S^{DBT}\Big|_{\delta_\beta=1}$, *then there exists* $\underline{\delta}_\beta > \delta_g$ *s.t. for all* $\delta_\beta > \underline{\delta}_\beta$, *we have that* $\Pi_S^{GSP}\Big|_{\delta_g} > \Pi_S^{DBT}\Big|_{\delta_\beta}$.

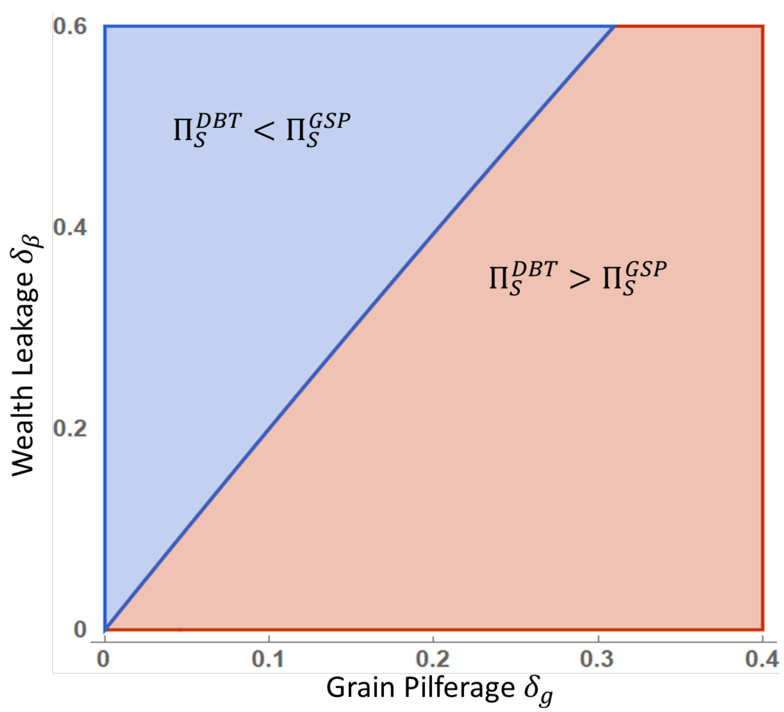


Figure 3 Comparison of the social planner's surplus under the DBT and the GSP scheme in the presence of leakages. The values of parameters used are: $M = 0.4, k = 4, n = 1, \theta = 0.9, \alpha = 0.25, b = 0.03, \beta = 0.01$. **In this case,** $b^*(\theta) \approxeq 0.25$.

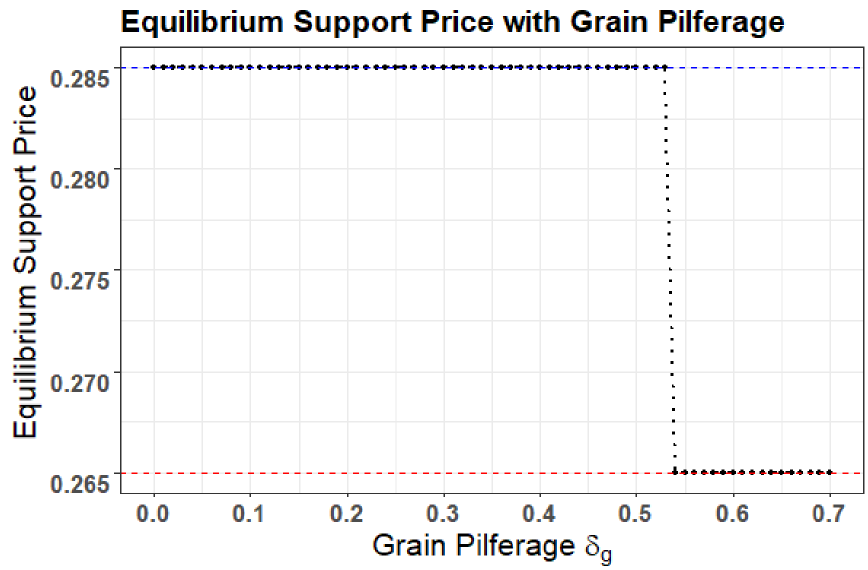
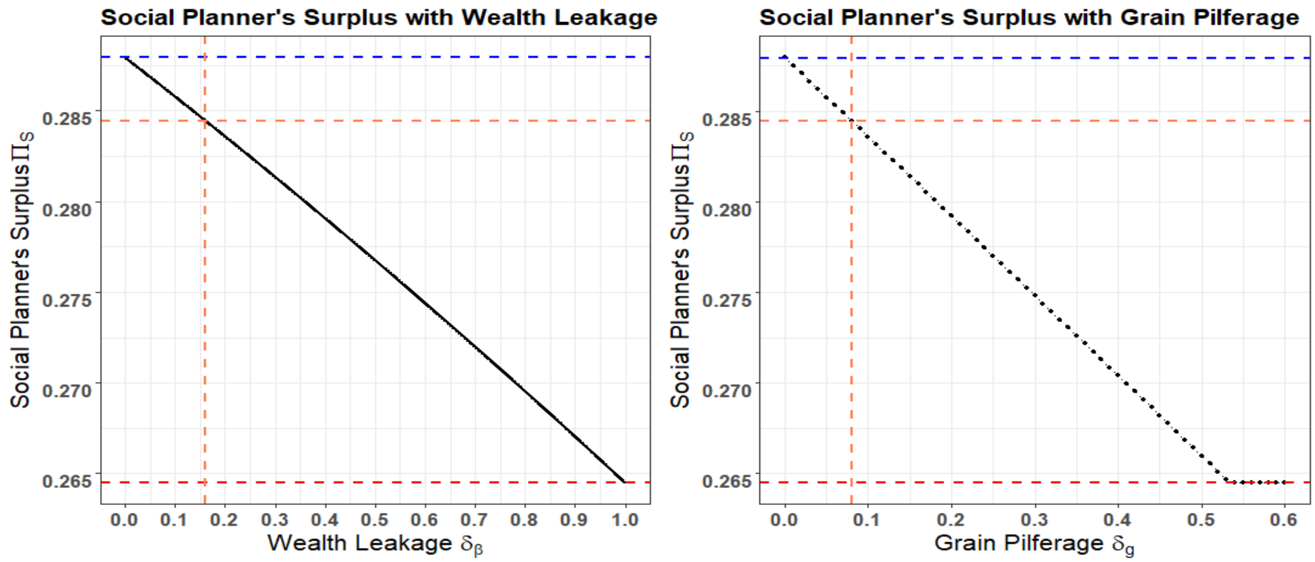


Figure 4 **Equilibrium support price with the extent of grain pilferage. Values of the parameters used for the simulation:** $M = 0.4, k = 4, n = 1, \theta = 0.9, \alpha = 0.25, b = 0.03, \beta = 0.01$. **In this case,** $p_m(1)^{NI} \approxeq 0.265$ **(red dotted line),** $p_m(1)^{DBT}\Big|_{\delta_\beta=0} = 0.285$ **(blue dotted line). In this case,** $\underline{\delta}_g = 0.53$.

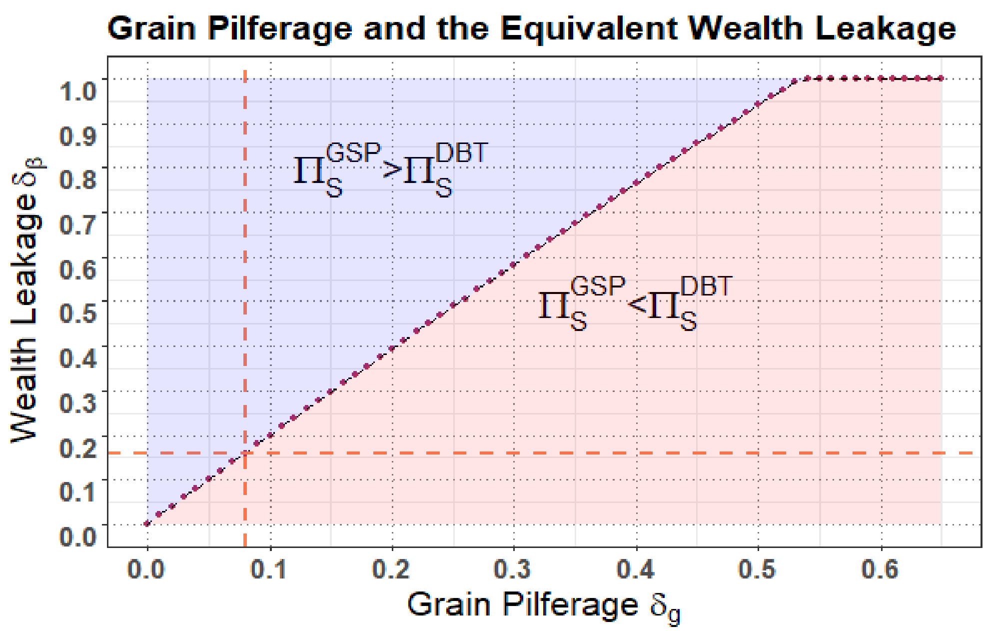
In words, if the magnitude of wealth leakage (δ_β) is sufficiently high, the social planner's surplus under the GSP scheme is higher than that under the DBT scheme. We provide an illustration in Figure 3: The blue region denotes outcomes where $\Pi_S^{GSP}\Big|_{\delta_g} > \Pi_S^{GSP}\Big|_{\delta_\beta}$.

Solving for $\underline{\delta}_g$ and $\underline{\delta}_\beta$ analytically is challenging. In Figure 4, we demonstrate, for a specific choice of parameters, the equilibrium support price of the social planner in the presence of grain pilferage as a function of δ_g. Observe that the equilibrium value of the support price is $p_m(1)^{DBT}\Big|_{\delta_\beta=0}$ if $\delta_g < \underline{\delta}_g = 0.53$. Beyond $\underline{\delta}_g$, the equilibrium support price is $p_m(1)^{NI}$; that is, the extent of grain pilferage is sufficiently high so that the social planner prefers the outcome under NI to the GSP scheme. The intuition is as follows: While the GSP scheme leads to an increase in the production of food grains, a fraction of the food grains is lost to pilferage. Recall that the production cost is convex in the production quantity, while the consumption utility is concave in the consumption quantity. Therefore, if the extent of pilferage is sufficiently high, then the production cost of food grains lost to pilferage is not offset by the net gain in the consumption due to greater production. Consequently, the social planner prefers the outcome under NI to the GSP scheme.

In Figure 5a, we numerically calculate the surplus of the social planner under the DBT scheme (as a function of wealth leakage) and the GSP scheme (as a function of grain pilferage). In Figure 5b, we demonstrate the following correspondence: Consider a fixed δ_g (extent of grain pilferage in the GSP scheme): We calculate $\underline{\delta}_\beta$ (the extent of wealth leakage in the DBT scheme) that leads to an identical surplus for the social planner. That is, $\Pi_S^{DBT}\Big|_{\underline{\delta}_\beta} = \Pi_S^{GSP}\Big|_{\delta_g}$. For instance, consider the



(a) Comparison of the social planner's surplus under the DBT scheme in the presence of wealth leakage (left) and the GSP scheme in the presence of grain pilferage (right). $\Pi_S^{DBT}\Big|_{\delta_\beta=0} \approxeq 0.287$ (blue dashed line), $\Pi_S^{DBT}\Big|_{\delta_\beta=1} \approxeq 0.264$ (red dashed line). For a fixed social planner's surplus (e.g., the orange dashed line), we indicate the extent of wealth leakage in the DBT scheme and grain leakage in the GSP scheme that achieve this surplus.



(b) The correspondence between a given value of grain pilferage δ_g under the GSP scheme and the value of wealth leakage δ_β under the DBT scheme that lead to identical surplus for the social planner. For instance, an 8% grain pilferage ($\delta_g = 0.08$) under the GSP scheme leads to the same surplus to the social planner as a 16% ($\delta_\beta = 0.16$) wealth leakage under the DBT scheme (indicated with the orange dashed line).

Figure 5 Values of the parameters used for the simulation: $M = 0.4$, $k = 4$, $n = 1$, $\theta = 0.9, \alpha = 0.25, b = 0.03, \beta = 0.01$.

GSP scheme with a grain pilferage of 8%. Figure 5b shows that the social planner's surplus under the GSP scheme is strictly larger than that under the DBT scheme if the value of wealth leakage exceeds 16%.

9. Conclusions

Broadly, our goal in this paper is twofold: (a) To understand the role of Guaranteed Support Prices (GSPs) on the operational decisions of its main stakeholders, viz., the farmers, the consuming population, and the social planner (government), and (b) To understand the impact of the GSP scheme on the welfare of each stakeholder and compare them with two benchmarks: (i) the absence of an intervention and (ii) the Direct Benefit Transfer (DBT) scheme.

Two key economic forces – poorness of the below-poverty-line (BPL) consumers (a demand-side impediment) and yield uncertainty (a supply-side impediment) – act as frictions to high production by the farmers and consumption by BPL consumers. We analyze two interventions (schemes) by the social planner: in-cash subsidies through the DBT scheme, where the social planner simply distributes cash to the BPL consumers, and in-kind subsidies through the GSP scheme, where the social planner procures foodgrains from the farmers and distributes them among the BPL consumers. In the absence of any leakages (specifically, *wealth leakage* in the DBT scheme and *grain pilferage* in the GSP scheme), we find that the performance of the DBT scheme is identical to that of the GSP scheme. Relative to the absence of an intervention if poorness is extreme, the GSP scheme leads to a strict improvement in the production by the farmers and the social planner's surplus. If yield uncertainty is dominant, the GSP scheme can be used as a mechanism to divide the budget of the social planner in any desired proportion to improve the surplus of the BPL consumers and the farmers.

For reasonable values of the leakages, the GSP scheme leads to a greater production by the farmers than the DBT scheme. The social planner's surplus under the GSP scheme is higher than that under the DBT scheme if the wealth leakage under the DBT scheme is sufficiently large. Finally, the GSP scheme also allows the social planner to maintain a reserve stock of foodgrains, thereby mitigating adverse effects of yield uncertainty and improving food security.

Our analysis focused on the GSP scheme for a single crop. In practice, many developing countries offer support prices for multiple crops; see, e.g., Planning Commission of India (2001). On the one hand, the government aims for crop-wise targets to "balance" their production quantities based on their relative demands from the consuming population; on the other hand, farmers – constrained by their respective geographical locations – have natural preferences over the crops but may get influenced in their choice via attractive support prices. This makes the analysis of farming effort and governmental decisions under multiple support prices a fairly complex problem. In particular, understanding the influence of support prices on the crop-mix pattern is an important and challenging problem for future research. A recent contribution along this direction is Chintapalli and Tang (2018), which considers the case of two crops.

A broader direction for future research is that of a comparison across different classes of subsidies. The World Trade Organization (WTO) classifies agricultural subsidies into *amber-box*, *blue-box*, and *green-box* subsidies (World Trade Organization 2018, The Guardian 2013): Amber-box subsidies involve support measures that can significantly distort production; the GSP scheme is one such subsidy. Green-box subsidies do not distort production at all, while blue-box subsidies only cause a moderate amount of distortion. The WTO regulates governmental spending by developing countries on these classes of subsidies – the current ceiling is 10% of the total value of agricultural production for amber-box subsidies and is 8% for blue-box subsidies. There is currently no cap on green-box subsidies, which include policies for environmental and regional protection. Thus, a comparative analysis of the tradeoffs between these classes of schemes can provide meaningful input to policymakers.

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Appendix A: Main Notation

Notation	Description
	Parameters
n	The size of the farming population ($n > 0$).
α	The production-cost parameter ($\alpha > 0$).
M	The size of the APL consumer segment ($M > 0$).
k	The size of the BPL consumer segment relative to
	the APL consumer segment ($k > 0$).
B	The budget of the social planner ($B > 0$).
b	The wealth of a BPL consumer ($b \geq 0$).
δ_r	The fraction of procured quantity withheld as reserve stock ($\delta_r \in [0,1]$).
δ_g	The extent of grain pilferage ($\delta_g \in [0,1]$).
δ_β	The extent of wealth leakage ($\delta_\beta \in [0,1]$).
	Decision Variables
p_g	The guaranteed support price (GSP).
q_e	The production effort exerted by a farmer.
q_m	The quantity sold by a farmer in the market
q_g	The quantity sold by a farmer to the social planner
q_{APL}	The quantity consumed by an APL consumer from the open-market.
q_{BPL}	The total quantity consumed by a BPL consumer.
	Other Variables
γ	Yield Realization
$\mathcal{Q}$	The total production effort by all farmers.
$p_m(\gamma)$	The market price under yield γ.
q_S	The quantity provided to a BPL consumer by the social planner (government).
	Scheme/Setting
NI	No Intervention.
DBT	Direct Benefit Transfer Scheme.
GSP	Guaranteed Support Price Scheme.

Table 1 **The main notation used in our analysis.**

We denote a variable of interest x under scheme/setting $t \in \{NI, DBT, GSP\}$ by x^t.

Appendix B: Proofs of Technical Results

Note: *To conserve notation in the technical proofs, we often suppress the arguments of functions when no confusion arises in doing so.*

Proof of Lemma 4.1: If $b \geq \frac{1}{4}$, then for any $p_m \in [0,1]$, we have that $0 \leq 1 - p_m \leq \frac{b}{p_m}$. Since $q_S = 0$, from (2) and (4), we have that $q^*_{BPL} = 1 - p_m$. Therefore, the total consumer demand in the open-market at a market price $p_m \in [0,1]$ is

$$D(p_m) = M(1+k)(1-p_m).$$

We assume that all farmers have the same yield realization γ and hold (identical) beliefs $\hat{p}_m(\gamma)$ about the market-price for any yield realization γ. From (9), their effort (given their beliefs about the market price) is given by

$$q^*_e = \frac{\mathbb{E}[\hat{p}_m(\gamma)\gamma]}{2\alpha}. \tag{32}$$

Suppose the realized yield is γ: The total quantity available in the open-market is given by $n\frac{\mathbb{E}[\hat{p}_m(\gamma)\gamma]\gamma}{2\alpha}$. Therefore, the market price (denoted by $p_m(\gamma)$) is obtained from (10) as follows:

$$M(1+k)(1-p_m(\gamma)) = \frac{n}{2\alpha}\,\mathbb{E}[\hat{p}_m(\gamma)\gamma]\gamma.$$

$$\Rightarrow p_m(\gamma) = 1 - \left(\frac{n}{2\alpha M(1+k)}\,\mathbb{E}[\hat{p}_m(\gamma)\gamma]\right)\gamma.$$

From (11), we have that $p_m(\gamma) = \hat{p}_m(\gamma)$. We substitute (11) in the above equation. Observe, from the equation above, that $p_m(\gamma)$ is linear in γ, with the intercept term equal to 1. Using this observation, we solve for $p_m(\gamma)$ and obtain the following:

$$p_m(\gamma) = 1 - \left(\frac{\frac{n}{2\alpha}\mu}{M(1+k) + \frac{n}{2\alpha}(\mu^2+\sigma^2)}\right)\gamma. \tag{33}$$

Substituting (33) in (32), we have

$$q^*_e = \frac{\mu}{2\alpha}\left(\frac{M(1+k)}{M(1+k) + \frac{n}{2\alpha}(\mu^2+\sigma^2)}\right).$$

■

Proof of Lemma 4.2: Consider a fixed value of $\theta \in [0,1]$. It is straightforward to see that $\hat{p}_m(0) = p_m(0) = 1$. We solve for the equilibrium value of $p_m(1)$ below. Using (9), we have that $q^*_e = \frac{\theta}{2\alpha}\hat{p}_m(1)$. Recall, from (14), that $b^*(\theta) = \frac{M(1+k)\frac{n\theta}{2\alpha}}{\left(M(1+k)+\frac{n\theta}{2\alpha}\right)^2}$. In the absence of an intervention, $q_S = 0$. Substituting (5) and (11) in (10), we have

$$M(1-p_m(1)) + kM\min\left\{1-p_m(1), \frac{b}{p_m(1)}\right\} = n\frac{\theta}{2\alpha}p_m(1). \tag{34}$$

Observe that the LHS of (34) is strictly decreasing in $p_m(1)$, while the RHS is strictly increasing in $p_m(1)$. At $p_m(1) = 1$, the LHS is strictly smaller than the RHS, while at $p_m(1) = 0$, the LHS is strictly larger than the RHS. Consequently, (34) has a unique solution for $p_m(1)$.

Depending on the value of b, one of the following occurs:

1. If $b > b^*(\theta)$, the solution to (34) is as follows:
$$p_m(1) = \frac{M(1+k)}{M(1+k) + \frac{n\theta}{2\alpha}}. \tag{35}$$
Further, at this value of $p_m(1)$, we have that $\frac{b}{p_m(1)} > 1 - p_m(1)$.
2. If $b < b^*(\theta)$, the solution to (34) is as follows:
$$p_m(1) = \frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}. \tag{36}$$
Further, at this value of $p_m(1)$, we have that $\frac{b}{p_m(1)} < 1 - p_m(1)$.

Using (36), (35), the observation that (34) has a unique solution for $p_m(1)$, and (32), we have
$$\begin{aligned} q_e^* &= \frac{\theta}{2\alpha} p_m(1) \\ \text{where } p_m(1) &= \begin{cases} \frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}, & \text{if } b > b^*(\theta); \\ \frac{M+\sqrt{M^2+4kMb(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}, & \text{if } b < b^*(\theta). \end{cases} \end{aligned}$$
■

Proof of Lemma 5.1: In this case, we have that $b + \beta < b^*(\theta)$. From Lemma 4.2 (or equivalently, (36)), recall that under NI, if $b < b^*(\theta)$, the equilibrium effort of a farmer is
$$q_e^* = \frac{\theta}{2\alpha}\left(\frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}\right),$$
and the equilibrium market price under high yield realization is
$$p_m(1) = \frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}.$$
Both these quantities are increasing in the wealth b of a BPL consumer. Consequently, under the DBT scheme, if $b + \beta < b^*(\theta)$, the production effort and the market-price are both higher. The equilibrium quantities are obtained by substituting $b \to (b + \beta)$ in the above expressions. Therefore, we have the first part of the result.

Under NI, the (expected) social planner's surplus under NI can be written as
$$\begin{aligned} {\Pi_S}^{NI} = & \underbrace{M\Big[\theta\Big(\int_0^{1-p_m(1)} (1-\xi)d\xi + (w_{APL} - p_m(1)(1-p_m(1)))\Big) + (1-\theta)(w_{APL})\Big]}_{\text{APL Consumers' Surplus}} \\ & \underbrace{+ kM\Big[\theta\Big(\int_0^{\frac{b}{p_m(1)}} (1-\xi)d\xi + \Big(b - p_m(1)\Big(\frac{b}{p_m(1)}\Big)\Big)\Big) + (1-\theta)(b)\Big]}_{\text{BPL Consumers' Surplus}} \\ & \underbrace{+ n\Big[\theta\Big(p_m(1)\frac{\theta}{2\alpha}p_m(1) - \alpha(\frac{\theta}{2\alpha}p_m(1))^2\Big) + (1-\theta)\Big(-\alpha\Big(\frac{\theta}{2\alpha}p_m(1)\Big)^2\Big)\Big]}_{\text{Farmers' Surplus}} \\ & + \underbrace{kM\beta}_{\text{Unused Budget}}, \end{aligned} \tag{37}$$
$$\text{where } p_m(1) = p_m(1)^{NI} = \frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}.$$

In (37), the first term represents the APL consumers' expected surplus, the second term represents the BPL consumers' expected surplus, the third term is the farmers' expected profit, and the fourth term is the unused budget of the social planner (which is the entire budget B under NI). We rewrite (37) as follows:

$$\begin{aligned}
{\Pi_S}^{NI} &= \underbrace{Mw_{APL} + kMb + kM\beta}_{\text{Wealth Across All Segments}} + \\
&\underbrace{\theta\left(M\int_0^{1-p_m(1)}(1-\xi-p_m(1))d\xi + kM\int_0^{\frac{b}{p_m(1)}}(1-\xi-p_m(1))d\xi\right)}_{\text{(Expected) Consumer Surplus}} + \\
&\underbrace{n\left(-\alpha\left(\frac{\theta}{2\alpha}p_m(1)\right)^2 + \theta p_m(1)\left(\frac{\theta}{2\alpha}p_m(1)\right)\right)}_{\text{(Expected) Farmers'/Producer Surplus}}, \qquad (38)
\end{aligned}$$

where the first term represents the total wealth available across both segments of the consuming population along with the budget of the social planner. The second and third terms are the consumption utilities from each segment of the consuming population. (38) can be further simplified as follows:

$$\Pi_S^{NI} = Mw_{APL} + kM(b+\beta) - \underbrace{n\alpha\left(\frac{\theta}{2\alpha}p_m(1)\right)^2}_{\text{Production Cost}} + \underbrace{\theta\left(M\int_0^{1-p_m(1)}(1-\xi)d\xi + kM\int_0^{\frac{b}{p_m(1)}}(1-\xi)d\xi\right)}_{\text{Consumption Utility}} \qquad (39)$$

That is, all transfers from the consumers to the farmers are internal – therefore, it is sufficient to consider the consumption utility of the consumers and the production cost of the farmers.

Under the DBT scheme, the social planner's surplus can be written as

$$\begin{aligned}
{\Pi_S}^{DBT} &= Mw_{APL} + kM(b+\beta) + \\
&M\Big[\theta\Big(\int_0^{1-p_m(1)}(1-\xi-p_m(1))d\xi\Big)\Big] + kM\Big[\theta\Big(\int_0^{\frac{b+\beta}{p_m(1)}}(1-\xi-p_m(1))d\xi\Big)\Big] + \\
&n\Big[-\alpha\Big(\frac{\theta}{2\alpha}p_m(1)\Big)^2 + \theta\Big(p_m(1)\big(\frac{\theta}{2\alpha}p_m(1)\big)\Big)\Big], \qquad (40)
\end{aligned}$$

where $p_m(1) = p_m(1)^{DBT} = \dfrac{M + \sqrt{M^2 + 4kM(b+\beta)(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}$.

Using a similar simplification as in the analysis of Π_S^{NI}, we can rewrite Π_S^{DBT} in (40) as follows:

$$\Pi_S^{DBT} = Mw_{APL} + kM(b+\beta) - n\alpha\left(\frac{\theta}{2\alpha}p_m(1)\right)^2 + \theta\left(M\int_0^{1-p_m(1)}(1-\xi)d\xi + kM\int_0^{\frac{b+\beta}{p_m(1)}}(1-\xi)d\xi\right) \qquad (41)$$

We show that the difference in the social planner's surplus, i.e., the difference in the RHS of (40) and (38) (or equivalently, the difference in the RHS of (41) and (39)) is strictly positive. Observe that the wealth available across all segments of the population ($Mw_{APL} + kMb + kM\beta$) comprises of internal monetary transfer among the agents. Consequently, it is sufficient to show that the derivative of the sum of the remaining three terms in (38) w.r.t. b is *strictly* positive. Let

$$\begin{aligned}
\check{\Pi}_S^{NI} &= M\Big[\theta\Big(\int_0^{1-p_m(1)}(1-\xi-p_m(1))d\xi\Big)\Big] + kM\Big[\theta\Big(\int_0^{\frac{b}{p_m(1)}}(1-\xi-p_m(1))d\xi\Big)\Big] + \\
&n\Big[-\alpha\Big(\frac{\theta}{2\alpha}p_m(1)\Big)^2 + \theta\Big(p_m(1)\big(\frac{\theta}{2\alpha}p_m(1)\big)\Big)\Big], \qquad (42)
\end{aligned}$$

$$\text{where } p_m(1) = p_m(1)^{NI} = \frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})};$$
$$\text{i.e., } \check{\Pi}_{SP}^{NI} = \Pi_{SP}^{NI} - (Mw_{APL} + kMb + kM\beta).$$

We will show that $\frac{d\check{\Pi}_{SP}^{NI}}{db} > 0$. We can write $\frac{d\check{\Pi}_{SP}^{NI}}{db}$ as

$$\frac{d\check{\Pi}_{S}^{NI}}{db} = \frac{\partial\check{\Pi}_{SP}^{NI}}{\partial b} + \frac{\partial\check{\Pi}_{SP}^{NI}}{\partial p_m(1)}\frac{\partial p_m(1)}{\partial b} \tag{43}$$

The first term represents the direct effect of an increase in b. The second term represents the indirect effect of the increase in b, because an increase in b leads to an increase in $p_m(1)$. Using Lemma 4.2, we have that:

$$\frac{\partial\check{\Pi}_{S}^{NI}}{\partial b} = \frac{kM\theta}{p_m(1)}[1 - p_m(1) - \frac{b}{p_m(1)}] > 0,$$
$$\frac{\partial\check{\Pi}_{S}^{NI}}{\partial p_m(1)} = -\frac{\theta kMb}{p_m(1)^2}[1 - p_m(1) - \frac{b}{p_m(1)}] < 0, \text{ and}$$
$$\frac{dp_m(1)}{db} = \frac{k}{2\sqrt{kb(1 + \frac{n\theta}{2M\alpha}) + \frac{1}{4}}},$$
$$\text{where } p_m(1) = \frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}.$$

The direct effect (of an increase in b) is positive, while the indirect effect is negative. Substituting these expressions back in (43) and using straightforward algebraic manipulations, we have the required result. ■

Proof of Lemma 6.1: Consider an announced p_g and farmers' belief $(\hat{p}_m(1), \hat{p}_m(0))$, where $\hat{p}_m(1) \le \hat{p}_m(0)$ and $\hat{p}_m(0) = 1$. Recall the farmer's decisions and objective from Section 3.2. One of the following holds:

(*i*) $p_g < \hat{p}_m(1)$, or

(*ii*) $p_g = \hat{p}_m(1)$, or

(*iii*) $p_g > \hat{p}_m(1)$.

For each of these cases, we solve the optimal selling decisions (q_g, q_m) for a fixed q_e when the realized yield is high (i.e., solution to `Problem` $\mathtt{P}_\mathtt{f}^2$). Then, we substitute this solution back into `Problem` $\mathtt{P}_\mathtt{f}^1$ to find the equilibrium q_e that maximizes the farmer's expected profit.

(*i*) Suppose $p_g < \hat{p}_m(1)$: Then, for any given q_e, it is straightforward that $q_g = 0$ and $q_m = q_e$ (i.e., the solution to `Problem` $\mathtt{P}_\mathtt{f}^2$). Substituting these solutions in `Problem` $\mathtt{P}_\mathtt{f}^1$, we have

$$\pi_f = -\alpha q_e^2 + \mathbb{E}[\hat{p}_m(\gamma)\gamma q_e]$$
$$\Rightarrow q_e^* = \frac{\mathbb{E}[\hat{p}_m(\gamma)\gamma]}{2\alpha} = \frac{\theta}{2\alpha}\hat{p}_m(1)$$

(*ii*) Suppose $p_g > \hat{p}_m(1)$: Then, for any given q_e, the solution to `Problem` $\mathtt{P}_\mathtt{f}^2$ is as follows:

$$q_g = \min\left\{\frac{B}{np_g}, q_e\right\} \text{ and}$$
$$q_m = q_e - q_g$$

Using these solutions, `Problem` $\mathtt{P}_\mathtt{f}^1$ can be reformulated as:

$$\pi_f(q_e) = -\alpha q_e^2 + \mathbb{E}\left[p_g \min\left\{q_e, \frac{B}{np_g}\right\} + \hat{p}_m(1)\left(q_e - \frac{B}{np_g}\right)^+\right],$$

$$q_e^* = \begin{cases} \frac{\theta}{2\alpha} p_g, & \frac{\theta}{2\alpha}\hat{p}_m(1) \le \frac{\theta}{2\alpha} p_g \le \frac{B}{np_g}; \\ \frac{B}{np_g}, & \frac{\theta}{2\alpha}\hat{p}_m(1) \le \frac{B}{np_g} \le \frac{\theta}{2\alpha} p_g; \\ \frac{\theta}{2\alpha}\hat{p}_m(1), & \frac{B}{np_g} \le \frac{\theta}{2\alpha}\hat{p}_m(1) \le \frac{\theta}{2\alpha} p_g \end{cases}$$

(iii) Suppose $p_g = \hat{p}_m(1)$: Then, for any given q_e, it is straightforward that any $(q_g, q_m) \ge 0$ s.t. $q_g + q_m = q_e$ is optimal (the solution to $\texttt{Problem P}_\texttt{f}^2$). Substituting these solutions in $\texttt{Problem P}_\texttt{f}^1$, we have

$$\pi_f(q_e) = -\alpha q_e^2 + \mathbb{E}[p_g \gamma q_e] \Rightarrow q_e^* = \frac{p_g\,\mathbb{E}[\gamma]}{2\alpha} = \frac{\theta}{2\alpha} p_g$$

■

Proof of Lemma 6.2: **(Production and Selling Decisions)** In this case, we have $b+\beta < b^*(\theta)$. Consider case 1, where the announced support price $p_g < p_m(1)^{NI}$. We show the result under this case in two steps:

(a) We show that the only rational belief of the farmers is $p_g < \hat{p}_m(1)$ and $\hat{p}_m(1) = p_m(1)^{NI}$.

(b) Using Lemma 6.1, we identify the equilibrium outcome.

We show (a) by contradiction. Suppose $p_g \not< \hat{p}_m(1)$: Then, either $p_g = \hat{p}_m(1)$ or $p_g > \hat{p}_m(1)$. Suppose $p_g = \hat{p}_m(1)$. Then, from Lemma 6.1, we have $q_e^* = \frac{\theta}{2\alpha} p_g$. Using (10), $p_g = \hat{p}_m(1)$, and decisions q_g and $q_m = q_e - q_g$, the market clearance condition is as follows:

$$M(1-p_g) + kM\min\left\{1 - p_g - \frac{n}{kM} q_g, \frac{b}{p_g}\right\} = n\left(\frac{\theta}{2\alpha} p_g - q_g\right). \tag{44}$$

We rewrite (44) as:

$$M(1-p_g) + kM\min\left\{1 - p_g, \frac{b}{p_g} + \frac{n}{kM} q_g\right\} = n\frac{\theta}{2\alpha} p_g. \tag{45}$$

All else equal, the LHS of (45) is decreasing in p_g and increasing in q_g. Since $p_g < p_m(1)^{NI}$, the LHS is strictly higher than $M(1-p_m(1)^{NI}) + kM\min\left\{1 - p_m(1)^{NI}, \frac{b}{p_m(1)^{NI}}\right\}$. Since the RHS of (45) is strictly increasing in p_g, we have that the RHS is strictly lower than $n\frac{\theta}{2\alpha} p_m(1)^{NI}$. However, recall the market clearance under NI if $b+\beta < b^*(\theta)$:

$$M(1-p_m(1)^{NI}) + kM\min\left\{1 - p_m(1)^{NI}, \frac{b}{p_m(1)^{NI}}\right\} = n\frac{\theta}{2\alpha} p_m(1)^{NI},$$

leading to a contradiction. Therefore, we eliminate the possible belief $p_g = \hat{p}_m(1)$. Using a similar argument, we eliminate the possible belief $p_g > \hat{p}_m(1)$. Therefore, the only rational beliefs are $p_g < \hat{p}_m(1)$.

We now solve for (b) (i.e., find the equilibrium outcome) below. Using the expressions in case (i) of Lemma 6.1, we have that:

$$q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1), q_g = 0, q_m = q_e$$

Substituting these and using (5), we obtain $\hat{p}_m(1) = p_m(1)^{NI}$.

The analysis under case 2 (where $p_g \in \left[p_m(1)^{NI}, p_m(1)^{DBT}\right]$) and case 3 (where $p_g > p_m(1)^{DBT}$) follow identical steps as above, where we first identify the unique equilibrium beliefs (on the market price) by eliminating impossible beliefs and then obtain the equilibrium outcome in the respective cases. In case 2 (where $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$), we show that the only rational belief of the farmers is $p_g = \hat{p}_m(1)$ (by eliminating the beliefs $p_g < \hat{p}_m(1)$ and $p_g > \hat{p}_m(1)$). Then, from Lemma 6.1, we have that $q_e = \frac{\theta}{2\alpha}p_g$, and any $(q_g, q_m) \geq 0$ such that $q_g + q_m = q_e$ is optimal. We focus on symmetric strategies (i.e., all farmers adopt the same strategy). Using algebraic manipulations, we have

$$M(1-p_g) + kM\min\left\{1-p_g, \frac{b}{p_g} + \frac{n}{kM}q_g\right\} = n\frac{\theta}{2\alpha}p_g.$$

Since $b+\beta < b^*(\theta)$ and $p_g \in \left[p_m(1)^{NI}, p_m(1)^{DBT}\right]$, we have that

$$\frac{b}{p_g} + \frac{n}{kM}q_g \leq 1 - p_g.$$

Therefore, the market clearance condition can be written as:

$$M(1-p_g) + kM\left(\frac{b}{p_g} + \frac{n}{kM}q_g\right) = n\frac{\theta}{2\alpha}p_g \implies q_g = \frac{\theta}{2\alpha}p_g - \frac{M}{n}(1-p_g) - \frac{kM}{n}\frac{b}{p_g}$$

In case 3, where $p_g > p_m(1)^{DBT}$, we show that the only rational belief is $p_g > \hat{p}_m(1)$ (by eliminating the beliefs $p_g \leq \hat{p}_m(1)$). Since $b+\beta < b^*(\theta)$ and $p_g > p_m(1)^{DBT}$, we have that:

$$\frac{B}{np_g} < \frac{\theta}{2\alpha}\hat{p}_m(1) < \frac{\theta}{2\alpha}p_g.$$

From Lemma 6.1,

$$q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1), q_g = \frac{B}{np_g}, q_m = q_e - q_g.$$

Using standard algebraic manipulations, we have that

$$M(1-\hat{p}_m(1)) + kM\min\left\{1-\hat{p}_m(1), \frac{b}{\hat{p}_m(1)} + \frac{\beta}{p_g}\right\} = n\left(\frac{\theta}{2\alpha}\hat{p}_m(1)\right).$$

Since $b+\beta < b^*(\theta)$, we have

$$M(1-\hat{p}_m(1)) + kM\left(\frac{b}{\hat{p}_m(1)} + \frac{\beta}{p_g}\right) = n\left(\frac{\theta}{2\alpha}\hat{p}_m(1)\right).$$

Therefore, we have that

$$\hat{p}_m(1) = \frac{\left(M + kM\frac{\beta}{p_g}\right) + \sqrt{\left(M + kM\frac{\beta}{p_g}\right)^2 + 4kMb\left(M + \frac{n\theta}{2\alpha}\right)}}{2\left(M + \frac{n\theta}{2\alpha}\right)}.$$

(Social Planner's Surplus) The analysis in case 1, where $p_g < p_m(1)^{NI}$ is straightforward: Using case 1 of the above result, we have that the market outcome in this case is identical to the outcome under NI. Therefore, the social planner's surplus is also identical to that under NI.

The social planner's surplus in case 2 (i.e., $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$) can be written as follows:

$$\Pi_S = (Mw_{APL} + kM(b+\beta)) - n\alpha q_e^{*2} + \theta\left(M\int_0^{1-p_g}(1-\xi)d\xi + kM\int_0^{\frac{1}{kM}\left(\frac{n\theta}{2\alpha}p_g - M(1-p_g)\right)}(1-\xi)d\xi\right),$$

where $q_e^* = \frac{\theta}{2\alpha}p_g$. The first derivative of Π_S w.r.t p_g can be written as

$$\frac{d\Pi_S}{dp_g} = \left(\frac{\theta(M + \frac{n\theta}{2\alpha})}{kM}\right)\left(M(1+k) + \frac{n\theta}{2\alpha}\right)\left(\frac{M(1+k)}{(M(1+k) + \frac{n\theta}{2\alpha})} - p_g\right).$$

The first two terms in the above expression are positive. The third term is the difference between the market price $p_m(1)^{NI}$ when $b > b^*(\theta)$ (i.e., the BPL consumers are effectively not budget constrained) and p_g and is decreasing in p_g. At $p_g = p_m(1)^{DBT}$, this difference is positive, since $b+\beta < b^*(\theta)$ and $p_m(1)^{NI}$ is increasing in b if $b < b^*(\theta)$. Therefore, at any value of $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$, this difference is positive. Hence $\frac{d\Pi_S}{dp_g} > 0$.

In case 3 where $p_g > p_m(1)^{DBT}$, the social planner's surplus is given by:

$$\Pi_S = (Mw_{APL} + kM(b+\beta)) - n\alpha {q_e^*}^2 + \theta\left(M\int_0^{1-p_m(1)}(1-\xi)d\xi + kM\int_0^{\frac{\beta}{p_g}+\frac{b}{p_m(1)}}(1-\xi)d\xi\right),$$

where

$$p_m(1) = \frac{\left(M + kM\frac{\beta}{p_g}\right) + \sqrt{\left(M + kM\frac{\beta}{p_g}\right)^2 + 4kMb\left(M + \frac{n\theta}{2\alpha}\right)}}{2\left(M + \frac{n\theta}{2\alpha}\right)}.$$

Observe that $p_m(1)$ is decreasing in p_g. The derivative of Π_S w.r.t p_g can be written as:

$$\frac{d\Pi_S}{dp_g} = (\theta kM)\left(\frac{d}{dp_g}\underbrace{\left(\frac{\beta}{p_g} + \frac{b}{p_m(1)}\right)}_{=q_{BPL}}\right)\left(\underbrace{1 - p_m(1)}_{=q_{APL}} - \underbrace{\left(\frac{\beta}{p_g} + \frac{b}{p_m(1)}\right)}_{=q_{BPL}}\right).$$

The first and third terms are positive, while the second term is negative. To see this, observe that the second term in the RHS is the derivative of the quantity consumed by a BPL consumer with p_g. Recall the market clearance condition:

$$M(1-p_m(1)) + kM\left(\frac{b}{p_m(1)} + \frac{\beta}{p_g}\right) = n\frac{\theta p_m(1)}{2\alpha}$$

Since $p_m(1)$ is decreasing in p_g, the quantity consumed by a BPL consumer $\left(\frac{b}{p_m(1)} + \frac{\beta}{p_g}\right)$ decreases in p_g. Therefore, the derivative of the quantity consumed by the BPL consumer w.r.t p_g this term is negative. The third term is positive, since $b+\beta < b^*(\theta)$. Therefore, $\frac{d\Pi_S}{dp_g} < 0$. ■

Proof of Lemma 6.3: The proof consists of three (mutually exclusive and exhaustive) cases, depending on the value of p_g: (a) $p_g < p_m(1)^{NI}$, (b) $p_g = p_m(1)^{NI}$, and (c) $p_g > p_m(1)^{NI}$. The approach is identical to the proof of Lemma 6.2. That is, for each case, we first show the only consistent beliefs on $\hat{p}_m(1)$ by eliminating impossible beliefs. Then, we find the equilibrium market outcome using Lemma 6.1.

Consider the case (a) where $p_g < p_m(1)^{NI}$ (resp., case (b) where $p_g = p_m(1)^{NI}$, and case (c) where $p_g > p_m(1)^{NI}$): Using an approach identical to the proof of Lemma 6.2, we can show that the only rational beliefs are $p_g < \hat{p}_m(1)$ (resp., $p_g = \hat{p}_m(1)$ and $p_g > \hat{p}_m(1)$) and that $\hat{p}_m(1) = p_m(1)^{NI}$. Further, using Lemma 6.1, we can show in each of the three cases that the market price $\hat{p}_m(1)$ is unique and is equal to $p_m(1)^{NI}$. A detailed proof is, therefore, avoided for brevity. ■

Proof of Theorem 8.1: Suppose the social planner distributes a fraction $(1-\delta_r)$ of the procured quantity, and the remainder is retained as reserve stock. We consider cases where δ_r is not too large. Recall the two cases we consider in our analysis: (a) $b+\beta < b^*(\theta)$, and (b) $b > b^*(\theta)$. First, we analyze the market outcome corresponding to any announced support price p_g in Lemma B.1 (the intermediate result below). Then, we calculate the social planner's surplus as a function of the announced support price. We show that for any p_g, $\frac{d\Pi_S}{d\delta_r} > 0$ if $f'(0)$ is sufficiently large.

Lemma B.1. *If the social planner distributes a fraction* $(1-\delta_r)$ *of the procured quantity, then the market price* $p_m(1)$ *depends on* b *as follows:*

(a) Suppose $b > b^*(\theta)$*:*

$$p_m(1) = \begin{cases} p_m(1)^{NI}, & p_g < p_m(1)^{NI}; \\ p_g, & p_g \in [p_m(1)^{NI}, \breve{p}_g]; \\ \frac{M(1+k)+\frac{B}{p_g}\delta_r}{M(1+k)+\frac{n\theta}{2\alpha}}, & p_g > \breve{p}_g. \end{cases}$$

where $\breve{p}_g$ *is the solution to* $p_g = \frac{M(1+k)+\frac{B}{p_g}\delta_r}{M(1+k)+\frac{n\theta}{2\alpha}}$.

(b) Suppose $b+\beta < b^*(\theta)$*:*

$$p_m(1) = \begin{cases} p_m(1)^{NI}, & p_g < p_m(1)^{NI}; \\ p_g, & p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]; \\ \frac{(M+\frac{B}{p_g})+\sqrt{(M+\frac{B}{p_g})^2+4bkM(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}, & p_g > p_m(1)^{DBT}. \end{cases}$$

Proof: In equilibrium, one of the following outcomes occur: (a) $p_g < p_m(1)$, (b) $p_g > p_m(1)$, and (c) $p_g = p_m(1)$.

Case (a): Suppose, in equilibrium, $p_g < p_m(1)$, we have that $q_g = 0$. Substituting this in the market clearance equation in (26), we have that the market outcome is identical to NI. Therefore, this case occurs if $p_g < p_m(1)^{NI}$. Recall the expression for $p_m(1)^{NI}$ from Lemma 4.2.

$$p_m(1)^{NI} = \begin{cases} \frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}, & b > b^*(\theta); \\ \frac{M+\sqrt{M^2+4kMb(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}, & b+\beta < b^*(\theta) \end{cases} \tag{46}$$

Therefore, if $b > b^*(\theta)$ and $p_g < \frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}$ or if $b+\beta < b^*(\theta)$ and $p_g < \frac{M+\sqrt{M^2+4kMb(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}$, then $p_g < p_m(1)^{NI}$. Under this case, $\Pi_S^{GSP} = \Pi_S^{NI}$ and is independent of δ_r.

Case (b): Suppose, in equilibrium, the following occurs: $p_g > p_m(1)$. Then, $q_g = \frac{B}{np_g}$. Substituting this in (26), the market clearance condition at $\gamma = 1$ is the following:

$$M(1-p_m(1)) + kM\min\left\{1-p_m(1)-\frac{\beta}{p_g}(1-\delta_r), \frac{b}{p_m(1)}\right\} = n\frac{\theta}{2\alpha}p_m(1) - \frac{B}{p_g}.$$

We have the following two cases:

1. Suppose $1-p_m(1)-\frac{\beta}{p_g}(1-\delta_r) \le \frac{b}{p_m(1)}$: Then,

$$p_m(1) = \frac{M(1+k)+\frac{B}{p_g}\delta_r}{M(1+k)+\frac{n\theta}{2\alpha}}.$$

Let $\breve{p}_g$ be the solution to $p_g = \frac{M(1+k)+\frac{B}{p_g}\delta_r}{M(1+k)+\frac{n\theta}{2\alpha}}$. Substituting this value of $p_m(1)$ back in the condition above, we have that this occurs if $b > b^*(\theta)$ and $p_g > \breve{p}_g$.

2. Otherwise, if $1-p_m(1)-\frac{\beta}{p_g}(1-\delta_r) > \frac{b}{p_m(1)}$, then

$$M(1-p_m(1)) + kM\frac{b}{p_m(1)} + \frac{B}{p_g} = n\frac{\theta}{2\alpha}p_m(1)$$

This is identical to the analysis in Section 6.2. We have

$$p_m(1) = \frac{(M+\frac{B}{p_g})+\sqrt{(M+\frac{B}{p_g})^2+4bkM(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}$$

Substituting this back in the condition above, this case arises if $b+\beta < b^*(\theta)$ and $p_g > p_m(1)^{DBT}$.

Case (c): Suppose, in equilibrium, the following outcome occurs: $p_g = p_m(1)$. Then, $q_g \in \left[0, \frac{B}{np_g}\right]$. Let $Q_g = nq_g$. Substituting in the market clearance condition:

$$M(1-p_m(1)) + kM \min\left\{1-p_m(1) - \frac{Q_g}{kM}(1-\delta_r), \frac{b}{p_m(1)}\right\} = n\frac{\theta}{2\alpha}p_m(1) - Q_g$$

We have the following two cases:

1. Suppose $1-p_m(1) - \frac{Q_g}{kM}(1-\delta_r) \leq \frac{b}{p_m(1)}$. From the market clearance condition above, we have:

$$p_g = p_m(1) = \frac{M(1+k)+Q_g\delta_r}{M(1+k)+\frac{n\theta}{2\alpha}}$$

Therefore, the unique symmetric strategy of the farmers leads to

$$Q_g = \frac{\left(M(1+k)+\frac{n\theta}{2\alpha}\right)p_g - M(1+k)}{\delta_r}.$$

Substituting this in the condition above, this case occurs if $b > b^*(\theta)$ and $\frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}} \leq p_g \leq \breve{p}$.

2. Suppose $1-p_m(1) - \frac{Q_g}{kM}(1-\delta_r) > \frac{b}{p_m(1)}$. As in the above case, the unique symmetric strategy of the farmers is s.t.

$$Q_g = \frac{n\theta}{2\alpha}p_g - M(1-p_g) - kM\frac{b}{p_g}.$$

This case occurs if $b+\beta < b^*(\theta)$ and $\frac{M+\sqrt{M^2+4kMb(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})} \leq p_g \leq \bar{p}_g$. Therefore, if $b+\beta < b^*(\theta)$, then:

$$p_m(1) = \begin{cases} p_m(1)^{NI}, & p_g < p_m(1)^{NI}; \\ p_g, & p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]; \\ \frac{(M+\frac{B}{p_g})+\sqrt{(M+\frac{B}{p_g})^2+4bkM(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}, & p_g > p_m(1)^{DBT}. \end{cases} \tag{47}$$

This expression is identical to the case when $\delta_r = 0$. If $b > b^*(\theta)$, then,

$$p_m(1) = \begin{cases} p_m(1)^{NI}, & p_g < p_m(1)^{NI}; \\ p_g, & p_g \in [p_m(1)^{NI}, \breve{p}_g]; \\ \frac{M(1+k)+\frac{B}{p_g}\delta_r}{M(1+k)+\frac{n\theta}{2\alpha}}, & p_g > \breve{p}_g. \end{cases} \tag{48}$$

■

The social planner's surplus can be written as:

$$\Pi_S = Mw_{APL} + kM(b+\beta) + n\frac{(\theta p_m(1))^2}{4\alpha} +$$
$$\theta\left(M\int_0^{q_{APL}}(1-p_m(1)-\xi)\,d\xi + kM\left(\int_0^{q_{BPL}+(1-\delta_r)\frac{Q_g}{kM}}(1-\xi)d\xi - p_m(1)q_{BPL}\right) - p_gQ_g + f(Q_g\delta_r)\right),$$

where the quantities purchased by an individual APL and BPL consumer are

$$q_{APL} = 1-p_m(1) \text{ and } q_{BPL} = \min\left\{1-p_m(1) - \frac{Q_g(1-\delta_r)}{kM}, \frac{b}{p_m(1)}\right\}$$

and Q_g is the total quantity procured by the social planner. We identify the conditions under which the equilibrium value of $I > 0$ under the two cases: (a) $b+\beta < b^*(\theta)$ and (b) $b > b^*(\theta)$.

(*a*) Suppose $b+\beta<b^*(\theta)$. In this case, $q_{BPL}=\frac{b}{p_m(1)}$. Substituting for the expressions for $p_m(1)$ and Q_g depending on the value of p_g from (47) in Π_S, we calculate $\frac{d\Pi_S}{d\delta_r}$:

$$\frac{d\Pi_S}{d\delta_r}=\begin{cases}\theta\left(a-b\delta_r+f'(Q_g\delta_r)Q_g\right), & \text{if } p_g\geq p_m(1)^{NI};\\ 0, & \text{o/w.}\end{cases}$$

where

$$a=\begin{cases}\frac{(\theta np_g-2\alpha M(k-p_g+1))(p_g(2\alpha M(p_g-1)+\theta np_g)-2\alpha bkM)}{4\alpha^2kMp_g}, & \text{if } p_g\in[p_m(1)^{NI},p_m(1)^{DBT}];\\ \frac{B\sqrt{\alpha\left(2bkMp_g^2(2\alpha M+\theta n)+\alpha(B+Mp_g)^2\right)}}{2\alpha kMp_g^2}+\frac{B^2}{2kMp_g^2}-\frac{B(2k+1)}{2kp_g}, & \text{if } p_g>p_m(1)^{DBT}.\end{cases}$$

and

$$b=\begin{cases}\frac{(p_g(2\alpha M(p_g-1)+\theta np_g)-2\alpha bkM)^2}{4\alpha^2kMp_g^2}, & \text{if } p_g\in[p_m(1)^{NI},p_m(1)^{DBT}];\\ \frac{B^2}{kMp_g^2}, & \text{if } p_g>p_m(1)^{DBT}.\end{cases}$$

For $\left.\frac{d\Pi_S}{d\delta_r}\right|_{\delta_r=0}>0$, we require that $f'(0)>-\frac{a}{Q_g}$. Using straightforward algebraic manipulation, we can show that a sufficient condition for $\left.\frac{d\Pi_S}{d\delta_r}\right|_{\delta_r=0}>0$ is:

$$f'(0)>\frac{M+\frac{n\theta}{2\alpha}}{kM}\left(\frac{M(1+k)}{M+\frac{n\theta}{2\alpha}}-p_m(1)^{NI}\right). \tag{49}$$

From (46), recall that if $b+\beta<b^*(\theta)$, then $p_m(1)^{NI}<\frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}$. Therefore, the second term in the RHS of (49) is strictly positive.

(*b*) Suppose $b>b^*(\theta)$. In this case, the expression for the social planner's welfare is as given above. In this case,

$$q_{BPL}=1-p_m(1)-\frac{Q_g(1-\delta_r)}{kM}.$$

Using a similar analysis, we have that if

$$f'(0)>\frac{M(1+k)-\frac{B}{p_m(1)^{NI}}}{M(1+k)+\frac{n\theta}{2\alpha}}. \tag{50}$$

holds, then $\left.\frac{d\Pi_S}{d\delta_r}\right|_{\delta_r=0}>0$. Using Assumptions 4.1 and 5.1 that $\frac{B}{p_m(1)^{NI}}<n\frac{\theta p_m(1)^{NI}}{2\alpha}<M(1+k)$. Therefore, the RHS is strictly positive.

■

Proof of Lemma 8.1: We demonstrate the conditions that must be satisfied for the support price $p_g=\left.p_m(1)^{DBT}\right|_{\delta_\beta=0}$. Recall the proof of Lemma 6.2 (the case of $\delta_g=0$): We have $\frac{d\Pi_S}{dp_g}>0$ if $p_g\in\left[p_m(1)^{NI},\left.p_m(1)^{DBT}\right|_{\delta_\beta=0}\right]$ and $\frac{d\Pi_S}{dp_g}<0$ if $p_g\in\left[\left.p_m(1)^{DBT}\right|_{\delta_\beta=0},\infty\right]$. Therefore, the equilibrium support price is $p_g=\left.p_m(1)^{DBT}\right|_{\delta_\beta=0}$. Now, suppose that $\delta_g>0$. For the equilibrium support price $p_g=\left.p_m(1)^{DBT}\right|_{\delta_\beta=0}$ (i.e., to remain unchanged), we require the same conditions as above to hold: $\frac{d\Pi_S}{dp_g}>0$ if $p_g\in\left[p_m(1)^{NI},\left.p_m(1)^{DBT}\right|_{\delta_\beta=0}\right]$ and $\frac{d\Pi_S}{dp_g}<0$ if $p_g\in\left[\left.p_m(1)^{DBT}\right|_{\delta_\beta=0},\infty\right]$. We demonstrate that if δ_g is sufficiently small, these conditions are satisfied. Since $b+\beta<b^*(\theta)$, the market clearance condition at $\gamma=1$ can be written as:

$$M(1-p_m(1))+kM\frac{b}{p_m(1)}=n\left(\frac{\theta}{2\alpha}p_m(1)-q_g\right),$$

where

$$q_g = \begin{cases} 0, & \text{if } p_g < p_m(1); \\ \min\{q_e, \frac{B}{np_g}\}, & \text{if } p_g > p_m(1); \\ \text{any } q_g \in [0, \min\{q_e, \frac{B}{np_g}\}], & \text{if } p_g = p_m(1). \end{cases}$$

Observe from the above market clearance condition that the total production and selling decisions of the farmers is independent of δ_g; consequently, the market-price $p_m(1)$ is also independent of δ_g. Consequently, corresponding to any announced support price, the total production quantity and market price are independent of δ_g.

- If $p_g \in \left[p_m(1)^{NI}, p_m(1)^{DBT}\Big|_{\delta_\beta=0}\right]$, we have that $p_m(1) = p_g$. The social planner's surplus can then be written as:

$$\begin{aligned} \Pi_S^{GSP}\Big|_{\delta_g} &= (Mw_{APL} + kMb + B) - n\alpha\left(\frac{\theta}{2\alpha}p_g\right)^2 + \\ &\quad \theta\left(M\int_0^{1-p_g}(1-x)dx + kM\int_0^{\frac{b}{p_g}+\frac{1}{kM}\left(n\frac{\theta p_g}{2\alpha}-M(1-p_g)-kM\frac{b}{p_g}\right)(1-\delta_g)}(1-x)dx\right) \\ &= \underbrace{(Mw_{APL} + kMb + B) - n\alpha\left(\frac{\theta}{2\alpha}p_g\right)^2 + \theta\left(M\int_0^{1-p_g}(1-x)dx + kM\int_0^{\frac{1}{kM}\left(n\frac{\theta p_g}{2\alpha}-M(1-p_g)\right)}(1-x)dx\right)}_{=\Pi_S^{GSP}\big|_{\delta_g=0}} + \\ &\quad \theta kM\int_{\frac{1}{kM}\left(n\frac{\theta p_g}{2\alpha}-M(1-p_g)\right)}^{\frac{1}{kM}\left(n\frac{\theta p_g}{2\alpha}-M(1-p_g)\right)+\delta_g\left(\frac{b}{p_g}-\frac{1}{kM}\left(n\frac{\theta p_g}{2\alpha}-M(1-p_g)\right)\right)}(1-x)dx \end{aligned}$$

  The first term in the RHS is $\Pi_S^{GSP}\Big|_{\delta_g=0}$. In Lemma 6.2, we showed that $\Pi_S^{GSP}\Big|_{\delta_g=0}$ is strictly increasing in p_g if $b+\beta < b^*(\theta)$. The first derivative of the second term in the RHS w.r.t p_g is 0 at $\delta_g = 0$. Thus, if δ_g is sufficiently small, the RHS is strictly increasing. Define the following: Let

$$\begin{aligned} \underline{\Delta}_{g1} &= \left\{\underline{\underline{\delta}}_g : \frac{d\Pi_S^{GSP}\Big|_{\delta_g}}{dp_g} > 0 \ \forall \delta_g \in [0, \underline{\underline{\delta}}_g)\right\} \\ \text{and } \underline{\delta}_{g1} &= \sup \underline{\Delta}_{g1} \end{aligned}$$

- If $p_g \in \left[p_m(1)^{DBT}\Big|_{\delta_\beta=0}, \infty\right)$, we have that $p_m(1)$ is as given in (21). The social planner's surplus can be written as:

$$\Pi_S^{GSP}\Big|_{\delta_g} = (Mw_{APL} + kMb + B) - n\alpha\left(\frac{\theta}{2\alpha}p_m(1)\right)^2 + \theta\left(M\int_0^{1-p_m(1)}(1-x)dx + kM\int_0^{\frac{b}{p_m(1)}+\frac{\beta}{p_g}(1-\delta_g)}(1-x)dx\right)$$

  As in the previous case, we can write

$$\Pi_S^{GSP}\Big|_{\delta_g} = \Pi_S^{GSP}\Big|_{\delta_g=0} - \theta kM\int_{\frac{b}{p_m(1)}+\frac{\beta(1-\delta_g)}{p_g}}^{\frac{b}{p_m(1)}+\frac{\beta}{p_g}}(1-x)dx$$

  From Lemma 6.2, the first term is strictly decreasing in p_g. The first derivative of the second term in the RHS w.r.t p_g at $\delta_g = 0$ is 0. Thus, if δ_g is sufficiently small, the RHS is strictly decreasing. Define the following:

$$\begin{aligned} \underline{\Delta}_{g2} &= \left\{\underline{\underline{\delta}}_g : \frac{d\Pi_S^{GSP}\Big|_{\delta_g}}{dp_g} < 0 \ \forall \delta_g \in [0, \underline{\underline{\delta}}_g)\right\}, \\ \text{and } \underline{\delta}_{g2} &= \sup \underline{\Delta}_{g2} \end{aligned}$$

Combining these two observations, we have that if $\delta_g \leq \underline{\delta}_g = \min\{\underline{\delta}_{g1}, \underline{\delta}_{g2}\}$, the equilibrium support price is $p_g = p_m(1)^{DBT}\Big|_{\delta_\beta=0}$. ■

Proof of Theorem 8.2: We compare the total production quantity under the two schemes in the presence of leakages below.

- DBT scheme in the presence of wealth leakage δ_β: The market clearance condition is given in (27). Let $\mathcal{Q}^{DBT}\Big|_{\delta_\beta}$ denote the total production under the DBT scheme with wealth leakage δ_β; $\mathcal{Q}^{DBT}\Big|_{\delta_\beta} = n\frac{\theta p_m(1)\big|_{\delta_\beta}}{2\alpha}$. Observe that the market clearance condition is identical to that under the DBT scheme, with $b+\beta$ replaced by $b+\beta(1-\delta_\beta)$. From Lemma 4.2, the equilibrium market price $p_m(1)$ is increasing in b. Therefore, we have that the market price in the presence of the wealth leakage is strictly lower than in its absence. Consequently, $\mathcal{Q}^{DBT}\Big|_{\delta_\beta}$ is decreasing in δ_β.
- GSP scheme in the presence of grain pilferage δ_g: The equilibrium support price $p_g = p_m(1)^{DBT}\Big|_{\delta_\beta=0}$. Consequently, $\mathcal{Q}^{GSP} = \mathcal{Q}^{DBT}\Big|_{\delta_\beta=0}$. That is, the production under the GSP scheme is independent of the extent of grain pilferage δ_g.

Since $\mathcal{Q}^{DBT}\Big|_{\delta_\beta}$ is decreasing in δ_β and $\mathcal{Q}^{GSP}(\delta_g) = \mathcal{Q}^{DBT}\Big|_{\delta_\beta=0}$ for all δ_g, we have that for any $\delta_\beta > 0$, $\mathcal{Q}^{DBT}(\delta_\beta) < \mathcal{Q}^{GSP}(\delta_g)$. ■

Proof of Theorem 8.3: Recall that $\Pi_S^{DBT}\Big|_{\delta_\beta}$ (resp., $\Pi_S^{GSP}\Big|_{\delta_g}$) is decreasing in δ_β (resp., δ_g) and $\Pi_S^{DBT}\Big|_{\delta_\beta=0} = \Pi_S^{GSP}\Big|_{\delta_g=0}$. Consider $\delta_g \in (0, \underline{\delta}_g)$.

First, if $\delta_\beta = \delta_g$, we have that $\Pi_S^{DBT}\Big|_{\delta_\beta} > \Pi_S^{GSP}\Big|_{\delta_g}$. To see this, observe that the $p_m(1)^{DBT}\Big|_{\delta_\beta} < p_m(1)^{GSP}\Big|_{\delta_g}$, i.e., the market price is lower under the DBT scheme. Therefore, the production cost of the farmers is lower while the APL and BPL consumer surplus is higher under the DBT scheme.

Second, Suppose $\Pi_S^{DBT}\Big|_{\delta_\beta=1} < \Pi_S^{GSP}\Big|_{\delta_g}$. Then, there exists $\underline{\delta}_\beta$ s.t. $\Pi_S^{DBT}\Big|_{\underline{\delta}_\beta} = \Pi_S^{GSP}\Big|_{\delta_g}$. Since $\Pi_S^{DBT}\Big|_{\delta_\beta}$ is decreasing in δ_β, we have that for all $\delta_\beta < \underline{\delta}_\beta$, $\Pi_S^{DBT}\Big|_{\delta_\beta} < \Pi_S^{DBT}\Big|_{\underline{\delta}_\beta} = \Pi_S^{GSP}\Big|_{\delta_g}$. ■

Appendix C: Analysis Under General Yield Distribution

C.1. Some Useful Results

We first demonstrate some general results that are useful in our analysis below. Consider a continuous random variable γ. The support of γ is $[0,1]$. Let $F(\cdot)$ (resp., $f(\cdot)$) denote the C.D.F (resp., p.d.f) of this distribution. Let x denote a variable of interest; $x \in [0,1]$. Corresponding to a realized γ, let x^γ denote the value of x. Let θ denote a parameter; $\theta \in [0,\infty]$. Let $a_\theta(x)$ and $b(x)$ denote two functions with the following properties:

(a) $a_\theta : [0,1] \mapsto [0,\infty]$, $b : [0,1] \to [0,\infty]$.

(b) $a_\theta(x)$ and $b(x)$ are continuous in x for any θ.

(c) $a_\theta(x)$ is strictly decreasing in x for a given θ, $a_\theta(x)$ is increasing in θ for a given x; $a_\theta(1) = 0$ for all $\theta \in [0,\infty]$.

(d) $b(x)$ is strictly increasing in x, $b(0) = 0$.

Consider a fixed θ. For any $\gamma \in [0,1]$, let x_θ^γ denote the solution to x^γ in the following equation:

$$a_\theta(x^\gamma) = \mathbb{E}[b(x^\gamma)]\gamma \tag{51}$$

Let $\mathcal{B}_\theta = \mathbb{E}[b(x_\theta^\gamma)]$. We show the following results.

Lemma C.1. *(a) Consider a fixed θ. x_θ^γ is decreasing in γ and is unique.*

(b) Consider a fixed $\gamma \in [0,1]$. x_θ^γ is increasing in θ.

Proof: (a) Consider a fixed θ. All else equal in (51) (including x^γ), the RHS is strictly increasing in γ. For the LHS to be equal to RHS, we require that x^γ be decreasing in γ. Next, consider a fixed γ. The LHS is strictly decreasing in x^γ, while the RHS is increasing in x^γ. Therefore, x^γ is unique. Since this holds for any value of γ, we have that x^γ is unique for any γ.

(b) Consider a fixed γ. Then $x^\gamma = a_\theta^{-1}(\mathcal{B}_\theta\gamma)$. Then

$$\begin{aligned} \mathbb{E}[b(x_\theta^\gamma)] &= \mathbb{E}[b(a_\theta^{-1}(\mathcal{B}_\theta\gamma))] \\ \Rightarrow \mathcal{B}_\theta &= \mathbb{E}_\gamma[b(a_\theta^{-1}(\mathcal{B}_\theta\gamma))] \end{aligned} \tag{52}$$

$\mathcal{B}_\theta$ is the fixed point to the above equation: the LHS is strictly increasing in $\mathcal{B}_\theta$, while the RHS is decreasing in $\mathcal{B}_\theta$ (since $a_\theta^{-1}(\cdot)$ is decreasing). Therefore, $\mathcal{B}_\theta$ is unique. Since $a_\theta(x)$ is increasing in θ, we have that the RHS is increasing in θ. The fixed point y is increasing in θ. Applying this result, we have that $\mathcal{B}_\theta$ is increasing in θ. Since $\mathcal{B}_\theta$ and $a_\theta^{-1}(\cdot)$ are increasing in θ, we have that x_θ^γ is increasing in θ. Alternatively, all else fixed in (51) (including x^γ), the LHS is increasing in θ. For the LHS to be equal to RHS, we require that x^γ is increasing in θ. ∎

From part (b) of the above result, we have the following.

Corollary C.1. *$\mathcal{B}_\theta$ is increasing in θ.*

We now analyze the outcomes under the three settings – NI, DBT and GSP – under a general yield distribution below. Consider an arbitrary distribution for the yield γ; $\gamma \in [0,1]$. Let $F(\cdot)$ (resp., $f(\cdot)$) denote the C.D.F (resp., p.d.f) of this distribution. We analyze below.

C.2. Absence of any Intervention

Recall the equilibrium production by the farmers in (8). We solve for the equilibrium outcome (i.e., the equilibrium market price and the production decisions of the farmers) as follows. From (9), (10) and (11), the market clearance condition can be rewritten as:

$$M(1-p_m(\gamma)) + kM\min\left\{1-p_m(\gamma), \frac{b}{p_m(\gamma)}\right\} = n\frac{\mathbb{E}[\gamma p_m(\gamma)]}{2\alpha}\gamma \tag{53}$$

LEMMA C.2. *All else equal, the equilibrium market price $p_m(\gamma)$ is decreasing in γ and is unique.*

Proof: This result follows from Lemma C.1. The LHS in (53) is (strictly) decreasing in $p_m(\gamma)$. All else equal (including $p_m(\gamma)$), the RHS in (53) is strictly increasing in γ. Therefore, for (53) to hold as an equality for any value of γ, we require that $p_m(\gamma)$ is decreasing in γ.
Next, consider a fixed value of γ. The LHS is (strictly) decreasing in $p_m(\gamma)$, while the RHS is increasing in $p_m(\gamma)$. Therefore, $p_m(\gamma)$ is unique. Since this holds for any value of γ, we have that $p_m(\gamma)$ is unique. ∎

In what follows, we provide a sketch for solving for the equilibrium outcome (i.e., the equilibrium production effort and the market price). From (9), (11) and Lemma C.2, we have that $\mathcal{Q}$ is unique. Then, (53) can be written as:

$$M(1-p_m(\gamma)) + kM\min\left\{1-p_m(\gamma), \frac{b}{p_m(\gamma)}\right\} = \mathcal{Q}\gamma \tag{54}$$

Define $\Gamma = \left\{\gamma : \frac{b}{p_m(\gamma)} \leq 1-p_m(\gamma)\right\}$, i.e., if $\gamma \in \Gamma$, then the BPL consumers are wealth-constrained. Since $p_m(\gamma) \in [0, 10$, observe that if $b > \frac{1}{4}$, then $\frac{b}{p_m(\gamma)} > 1-p_m(\gamma)$. Therefore, a necessary condition is $b \leq \frac{1}{4}$ for this case to occur. We solve for the market price in (54).

$$p_m(\gamma) = \frac{(M-\mathcal{Q}\gamma) + \sqrt{4bkM^2 + (M-\mathcal{Q}\gamma)^2}}{2M}$$

Let $\Gamma^{\complement} = [0,1] \setminus \Gamma$, i.e., $\Gamma^{\complement} = \left\{\gamma : \frac{b}{p_m(\gamma)} > 1-p_m(\gamma)\right\}$, i.e., if $\gamma \in \Gamma^{\complement}$, the BPL consumers are not wealth-constrained. Then, we solve for the market price in (54).

$$p_m(\gamma) = 1-\gamma\left(\frac{\mathcal{Q}}{M(1+k)}\right).$$

Define

$$\begin{aligned} \gamma_- &= \frac{M(1+k)}{2\,\mathcal{Q}}\left(1-\sqrt{1-4b}\right), \text{and} \\ \gamma_+ &= \min\left\{1, \frac{M(1+k)}{2\,\mathcal{Q}}\left(1+\sqrt{1-4b}\right)\right\}. \end{aligned} \tag{55}$$

Then, $\Gamma = [\gamma_-, \gamma_+]$ and $\Gamma^{\complement} = [0,1] \setminus \Gamma$.[14] Therefore,

$$p_m(\gamma) = \begin{cases} 1-\gamma\left(\frac{\mathcal{Q}}{M(1+k)}\right) & \text{if } \gamma \in \Gamma^{\complement} \\ \frac{M-\mathcal{Q}\gamma+\sqrt{2bkM^2+(M-\mathcal{Q}\gamma)^2}}{2M} & \text{if } \gamma \in \Gamma \end{cases} \tag{56}$$

[14] If $b \geq \frac{1}{4}$, then $\Gamma^{\complement} = [0,1]$.

That is, $p_m(\gamma)=\min\left\{1-\gamma\left(\frac{\mathcal{Q}}{M(1+k)}\right),\frac{M-\mathcal{Q}\gamma+\sqrt{2bkM^2+(M-\mathcal{Q}\gamma)^2}}{2M}\right\}$. Recall from (9) and (11), $\mathcal{Q}=n\frac{\mathbb{E}[p_m(\gamma)\gamma]}{2\alpha}$. Combining with (56), we have

$$\mathcal{Q}=\frac{n}{2\alpha}\mathbb{E}_\gamma\left[\min\left\{1-\gamma\left(\frac{\mathcal{Q}}{M(1+k)}\right),\frac{M-\mathcal{Q}\gamma+\sqrt{2bkM^2+(M-\mathcal{Q}\gamma)^2}}{2M}\right\}\right]. \tag{57}$$

That is,

$$\mathcal{Q}=\frac{n}{2\alpha}\left(\int_{\gamma\in\Gamma^\complement}\left(1-\gamma\left(\frac{\mathcal{Q}}{M(1+k)}\right)\right)\gamma f_\gamma(\gamma)d\gamma+\int_{\gamma\in\Gamma}\left(\frac{(M-\mathcal{Q}\gamma)+\sqrt{2bkM^2+(M-\mathcal{Q}\gamma)^2}}{2M}\right)\gamma f_\gamma(\gamma)d\gamma\right), \tag{58}$$

where $f_\gamma(\gamma)$ denotes the p.d.f of γ. The total production by the farmers in equilibrium under NI solves (58).

Lemma C.3. *A unique value for $\mathcal{Q}\in\left(0,\frac{n}{2\alpha}\right)$ solves (58).*

Proof: It is sufficient to show that (57) has a unique solution for $\mathcal{Q}$. The LHS is strictly increasing in $\mathcal{Q}$ while the RHS is decreasing in $\mathcal{Q}$. At $\mathcal{Q}=0$, the LHS is strictly less than the RHS, while at $\mathcal{Q}=\frac{n}{2\alpha}$, the LHS is strictly higher than the RHS. Therefore, (57) has a unique solution for $\mathcal{Q}$.

Lemma C.4. *All else equal, if $b\le\frac{1}{4}$, $\mathcal{Q}$ is increasing in b. If $b>\frac{1}{4}$, then, the total production effort of the farmers is independent of b.*

Proof: This result follows from Corollary C.1. If $b\le\frac{1}{4}$, all else equal (including $\mathcal{Q}$), the RHS in (58) (or equivalently, (57)) is increasing in b. This is because $p_m(\gamma)$ is increasing in b if $\gamma\in\Gamma$, γ_- (resp., γ_+) is increasing (resp., decreasing) in b and $p_m(\gamma)$ is continuous. For the LHS to be equal to RHS, we require that $\mathcal{Q}$ is increasing in b. If $b>\frac{1}{4}$, then $\Gamma^\complement=[0,1]$. Therefore, the RHS is independent of b and hence $\mathcal{Q}$ is independent of b.

In general, solving for $\mathcal{Q}$ in (58) in closed-form is difficult due to the second term in the RHS.

Social Planner's Surplus Recall from Section 3.3 that the social planner's surplus under NI is

$$\Pi_S=(Mw_{APL}+kMb+B)+\underbrace{\mathbb{E}_\gamma\left[M\int_0^{1-p_m(\gamma)}(1-p_m(\gamma)-\xi)\,d\xi+kM\int_0^{\min\{1-pm(\gamma),\frac{b}{p_m(\gamma)}\}}(1-p_m(\gamma)-\xi)\,d\xi\right]}_{\text{Consumer Surplus}}+\underbrace{n\pi_f}_{\text{Producer Surplus}}$$

Since all monetary payments are transfers, we can rewrite this as

$$\Pi_S=(Mw_{APL}+kMb+B)-\frac{\alpha}{n}\mathcal{Q}^2+\underbrace{\mathbb{E}_\gamma\left[M\int_0^{1-p_m(\gamma)}(1-\xi)d\xi+kM\int_0^{\min\left\{1-pm(\gamma),\frac{b}{p_m(\gamma)}\right\}}(1-\xi)d\xi\right]}_{\text{Utility from consumption}} \tag{59}$$

Let $u_C(q)=\int_0^q(1-\xi)d\xi$ denote the utility from consumption for any consumer. Below, we analyze the social planner's surplus with an increase in b.

Theorem C.1. *All else equal, if $b<\frac{1}{4}$, Π_S^{NI} is increasing in b if $b<\frac{1}{4}$ and is independent of b otherwise.*

Proof: Consider the first derivative of Π_S w.r.t b from (62).

$$\frac{d\Pi_S}{db} = -\frac{\mathcal{Q}}{n/2\alpha}\frac{d\,\mathcal{Q}}{db} + \mathbb{E}_\gamma\left[Mu'_C(q_{APL}(\gamma))\frac{dq_{APL}(\gamma)}{db} + kMu'_C(q_{BPL}(\gamma))\frac{dq_{BPL}(\gamma)}{db}\right]$$

Since $q_{BPL}(\gamma) \le q_{APL}(\gamma)$, we can rewrite this equation as:

$$\frac{d\Pi_S}{db} = \mathbb{E}_\gamma\left[\left(u'_C\left(q_{BPL}(\gamma)\right) - u'_C\left(q_{APL}(\gamma)\right)\right)\frac{d(kMq_{BPL}(\gamma))}{db}\right] + \frac{d\,\mathcal{Q}}{db}\mathbb{E}_\gamma\left[u'_C(q_{APL}(\gamma)) - \frac{\mathcal{Q}}{n/2\alpha}\right]$$

We show that each of the terms in the RHS above are non-negative.
Consider the first term on the RHS in the equation above. Since $q_{BPL}(\gamma) \le q_{APL}(\gamma)$ and $q_{BPL}(\gamma)$ is increasing in b, the first term is non-negative. In particular, $q_{BPL}(\gamma) < q_{APL}(\gamma)$ if $b < \frac{1}{4}$ and $f(\gamma) > 0$ for some $\gamma \in \Gamma$. Therefore, the first term is strictly positive is $b < \frac{1}{4}$.
Consider the term inside the expectation in the second term. We have that $u'_C(q_{APL}(\gamma)) = p_m(\gamma)$. Substituting for this, the second term is as follows:

$$\begin{aligned} p_m(\gamma) - \frac{\mathcal{Q}}{n/2\alpha} &\ge p_m(\gamma)\gamma - \frac{\mathcal{Q}}{n/2\alpha} \\ \Rightarrow \mathbb{E}_\gamma\left[p_m(\gamma) - \frac{\mathcal{Q}}{n/2\alpha}\right] &\ge \mathbb{E}\left[p_m(\gamma)\gamma - \frac{\mathcal{Q}}{n/2\alpha}\right] = 0. \end{aligned}$$

Therefore, $\frac{d\Pi_S}{db} \ge 0$ and this condition holds as a strict inequality if $b < \frac{1}{4}$. ■

C.3. Direct Benefit Transfer Scheme

For any given b and β, one of the following occurs:

(a) $b + \beta \le \frac{1}{4}$,

(b) $b < \frac{1}{4} \le b + \beta$, or

(c) $\frac{1}{4} \le b$.

Since β is small relative to b, we ignore case (b) and focus on cases (a) and (c). Recall, from Lemma C.4, that the effect of an increase in b depends on whether $b \le \frac{1}{4}$ or not. Recall from Footnote 14 that if $b \ge \frac{1}{4}$, then for any realized yield γ, we have that $\frac{b}{p_m(\gamma)} \ge 1 - p_m(\gamma)$. That is, the BPL consumers are never wealth constrained.

The analysis in this section is identical to the analysis in Section C.2, except that b is replaced with $b + \beta$. Recall the definitions of γ_- and γ_+ from (55). Under the DBT scheme, we have

$$\begin{aligned} \gamma_- &= \frac{M(1+k)}{2\,\mathcal{Q}}\left(1 - \sqrt{1 - 4(b+\beta)}\right) \\ \gamma_+ &= \min\left\{1, \frac{M(1+k)}{2\,\mathcal{Q}}\left(1 + \sqrt{1 - 4(b+\beta)}\right)\right\} \end{aligned}$$

Recall the definition of $\Gamma = [\gamma_-, \gamma_+]$ (resp., $\Gamma^\complement = [0,1] \setminus \Gamma$). Analogous to (56), the market price corresponding to a realized yield γ under DBT can be written as:

$$p_m(\gamma) = \begin{cases} 1 - \gamma\left(\frac{\mathcal{Q}}{M(1+k)}\right) & \text{if } \gamma \in \Gamma^\complement \\ \frac{M - \mathcal{Q}\gamma + \sqrt{2(b+\beta)kM^2 + (M - \mathcal{Q}\gamma)^2}}{2M} & \text{if } \gamma \in \Gamma \end{cases} \tag{60}$$

Recall the equilibrium production quantity under NI from (58). Under DBT, the equilibrium production quantity can be written as:

$$\begin{aligned}
\mathcal{Q} &= \frac{n}{2\alpha}\mathbb{E}_\gamma\left[\min\left\{1-\gamma\left(\frac{\mathcal{Q}}{M(1+k)}\right), \frac{(M-\mathcal{Q}\gamma)+\sqrt{4(b+\beta)kM^2+(M-\mathcal{Q}\gamma)^2}}{2M}\right\}\right], \text{ i.e.,} \\
\mathcal{Q} &= \frac{n}{2\alpha}\mathbb{E}_\gamma\Big[\int_{\gamma\in\Gamma^c}\left(1-\gamma\left(\frac{\mathcal{Q}}{M(1+k)}\right)\right)f_\gamma(\gamma)d\gamma + \\
&\quad \int_{\gamma\in\Gamma}\frac{(M-\mathcal{Q}\gamma)+\sqrt{4(b+\beta)kM^2+(M-\mathcal{Q}\gamma)^2}}{2M}f_\gamma(\gamma)d\gamma\Big],
\end{aligned} \tag{61}$$

(61) is identical to (58), except that b is replaced with $b+\beta$: As before, the LHS in (61) is strictly increasing in $\mathcal{Q}$, while the RHS is decreasing in $\mathcal{Q}$, and therefore, (61) has a unique solution for $\mathcal{Q}$.

Comparison with NI Recall from Lemma C.4 that $\mathcal{Q}$ is increasing in b if $b<\frac{1}{4}$ and remains a constant thereafter. Under DBT, the wealth of a BPL consumer is $b+\beta$ $(>b)$. As a consequence of Lemma C.4, we have the following result that compares the total production by the farmers under DBT and NI.

COROLLARY C.2. *If the BPL consumers are wealth-constrained (i.e., $b+\beta\le\frac{1}{4}$), then $\mathcal{Q}^{DBT}>\mathcal{Q}^{NI}$. Otherwise, if $b\ge\frac{1}{4}$, the total production is independent of b and hence $\mathcal{Q}^{DBT}=\mathcal{Q}^{NI}$.*

Social Planner's Surplus The social planner's surplus can be written as:

$$\begin{aligned}
&\Pi_S = (Mw_{APL}+kM(b+\beta)) + \\
&\underbrace{\mathbb{E}_\gamma\left[M\int_0^{1-p_m(\gamma)}(1-p_m(\gamma)-\xi)\,d\xi + kM\int_0^{\min\left\{1-pm(\gamma),\frac{b+\beta}{p_m(\gamma)}\right\}}(1-p_m(\gamma)-\xi)\,d\xi\right]}_{\text{Consumer Surplus}} + \underbrace{n\pi_f}_{\text{Producer Surplus}}
\end{aligned} \tag{62}$$

Observe that all monetary payments (between consumers and farmers or the social planner and farmers) are internal transfers.

$$\Pi_S = (Mw_{APL}+kMb+B) - n\alpha q_e^2 + \mathbb{E}_\gamma\left[M\int_0^{1-p_m(\gamma)}(1-\xi)\,d\xi + kM\int_0^{\min\left\{1-pm(\gamma),\frac{b+\beta}{p_m(\gamma)}\right\}}(1-\xi)\,d\xi\right]$$

From Theorem C.1, we have the following result.

THEOREM C.2. *If $b+\beta<\frac{1}{4}$, then $\Pi_S^{DBT}>\Pi_S^{NI}$. If $b\ge\frac{1}{4}$, then $\Pi_S^{DBT}=\Pi_S^{NI}$.*

C.4. Guaranteed Support Price Scheme

The sequence of events are as discussed in Section 3. We solve and analyze the market outcome under a general yield distribution. Consider an announced support price p_g. We begin by analyzing the subgame corresponding to this announced support price.

Production and Selling Decisions Denote the farmers' beliefs on the market price by $\hat{p}_m(\gamma)$. Consider a representative farmer. Let q_e denote his production decision. Corresponding to any realized yield γ, the farmer's (anticipated) selling decisions are as follows:

1. $p_g<\hat{p}_m(\gamma)$: $q_g=0$, $q_m=q_e\gamma$.
2. $p_g>\hat{p}_m(\gamma)$: $q_g=\min\{\frac{B}{np_g},q_e\gamma\}$, $q_m=q_e\gamma-q_g=\max\{0,q_e\gamma-\frac{B}{np_g}\}$

3. $p_g = \hat{p}_m(\gamma)$: Any $q_g \in \left[0, \min\left\{q_e\gamma, \frac{B}{np_g}\right\}\right]$, $q_m = q_e\gamma - q_g$.

The farmer's expected profit can be written as

$$\pi_f = -\underbrace{\alpha q_e^2}_{=c(q_e)} + \mathbb{E}_\gamma\left[\underbrace{\hat{p}_m(\gamma)q_e\gamma + (p_g - \hat{p}_m(\gamma))\,1_{\{p_g > \hat{p}_m(\gamma)\}}\min\left\{\frac{B}{np_g}, q_e\gamma\right\}}_{=r^\gamma(q_e)}\right]. \tag{63}$$

The term inside the expectation is the farmer's revenue under yield γ, denoted by $r^\gamma(q_e)$. Then, $\pi_f(q_e) = -c(q_e) + \mathbb{E}_\gamma[r^\gamma(q_e)]$.

Lemma C.5. *$\pi_f(q_e)$ is concave in q_e.*

Proof: $c(q_e)$ is convex in q_e. We show that $r^\gamma(q_e)$ is concave in q_e. Rewrite $r^\gamma(q_e)$ as follows:

$$r^\gamma(q_e) = (\hat{p}_m(\gamma)\gamma)\,q_e + (p_g - \hat{p}_m(\gamma))\,1_{\{p_g > \hat{p}_m(\gamma)\}}\min\left\{\frac{B}{np_g}, q_e\gamma\right\}$$

In the RHS, the first term is linear in q_e, while the second is concave in q_e. Therefore, the RHS is concave in q_e. Since $r^\gamma(q_e)$ is concave in q_e, $\mathbb{E}[r^\gamma(q_e)]$ is also concave in q_e; therefore, $\pi_f(q_e)$ is concave in q_e. ■

Since $\pi_f(q_e)$ is concave in q_e, we apply first order conditions to solve for the optimal q_e. That is,

$$\begin{aligned}
\pi_f'(q_e) &= 0 \implies c'(q_e) = \mathbb{E}_\gamma[r^{\gamma\prime}(q_e)] \\
\Rightarrow 2\alpha q_e &= \mathbb{E}\left[\hat{p}_m(\gamma)\gamma + (p_g - \hat{p}_m(\gamma))\gamma\,1_{\left\{p_g > \hat{p}_m(\gamma), \gamma q_e < \frac{B}{np_g}\right\}}\right].
\end{aligned} \tag{64}$$

Since farmers are homogenous, their beliefs on the market prices are identical, and hence their production and selling decisions are identical. Let $\mathcal{Q} = nq_e$ denote the total production by the farmers.

Consumption Decisions Suppose the social planner provides a quantity q_S for consumption to each BPL consumer. Using (4), we can write the demand in the open-market as

$$D(p_m) = M(1 - p_m) + kM\min\left\{1 - q_S - p_m, \frac{b}{p_m}\right\}.$$

Suppose each farmers sell q_g to the social planner and the social planner distributes all of the procured quantity among the BPL consumers. Then, $q_S = \frac{n}{kM}q_g$. Rewriting the above equation,

$$D(p_m) = M(1 - p_m) + kM\min\left\{1 - \frac{n}{kM}q_g - p_m, \frac{b}{p_m}\right\}.$$

Equilibrium Analysis Suppose the realized yield is γ. Then, farmers anticipate a market price $\hat{p}_m(\gamma)$ and decide the quantities (q_g, q_m) to sell in each channel. Similar to (10) and (11), the following should hold in equilibrium:

$$\begin{aligned}
D(p_m(\gamma)) &= Q\gamma - nq_g, \text{ and} \\
\hat{p}_m(\gamma) &= p_m(\gamma)
\end{aligned} \tag{65}$$

However, Assumption 5.1 is not sufficient. We require the stronger assumption below.

Assumption C.1. *For any support price p_g and realized yield γ such that the farmers sell a strictly positive quantity to the social planner, the budget of the social planner is less than the total value of foodgrains at the support price, i.e., $B < p_g \gamma \mathcal{Q}$.*

Using Assumption C.1, we can rewrite (64) as $q_e = \frac{\mathbb{E}[p_m(\gamma)\gamma]}{2\alpha}$. Then, one of the following three cases arises:
Case 1: Suppose, in equilibrium, the following condition holds: $p_g < p_m(\gamma)$ *for all* γ. In this case, $q_g = 0$. Rewriting the market clearance condition,

$$M(1 - p_m(\gamma)) + kM \min\left\{1 - p_m, \frac{b}{p_m}\right\} = \left(n\frac{\mathbb{E}[p_m(\gamma)\gamma]}{2\alpha}\right)\gamma$$

Lemma C.6. *If $p_g < p_m(1)^{NI}$, the market outcome under GSP is identical to that under NI. Consequently, $\mathcal{Q}^{GSP} = \mathcal{Q}^{NI}$.*

Proof: The market clearance above is identical to (53). Therefore, the market outcome is identical to that under NI. The market price $p_m(\gamma)$ is decreasing in γ (from Lemma C.2) and hence $p_g < p_m(\gamma)$ for all values of γ is equivalent to $p_g < p_m(1)^{NI}$. ■

Case 2: Suppose, in equilibrium, the following condition holds: $p_g > p_m(\gamma)$ *for all* values of γ. In this case, $q_g = \min\{\gamma q_e, \frac{B}{np_g}\}$. Due to Assumption C.1, $q_g = \frac{B}{np_g}$. Substituting this in (65), the market clearance condition in (65) can be succinctly written as:

$$M\left(1 - p_m(\gamma)\right) + kM \min\left\{1 - p_m(\gamma), \frac{b}{p_m(\gamma)} + \frac{\beta}{p_g}\right\} = n\frac{\mathbb{E}\left[p_m(\gamma)\gamma\right]}{2\alpha}\gamma \tag{66}$$

Lemma C.7. *All else equal, $p_m(\gamma)$ is decreasing in γ and is unique.*

Proof: Consider a fixed $\mathcal{Q}$. The RHS is increasing in γ, while the LHS is decreasing in $p_m(\gamma)$. Therefore, $p_m(\gamma)$ is decreasing in γ. Next, for a fixed γ, the RHS is increasing in $p_m(\gamma)$, while the LHS is decreasing in $p_m(\gamma)$. Therefore, $p_m(\gamma)$ is unique. Since this holds for any value of γ, we have that $p_m(\gamma)$ is unique. ■

For $p_g > p_m(\gamma)$ to hold for all values of γ, from Lemma C.7, it suffices $p_g > p_m(0) = 1$. Therefore, this case occurs if $p_g > 1$.

To summarize this discussion, if $p_g > 1$, then $p_g > p_m(\gamma)$ for all values of γ. Next, we compare the total production by the farmers for different values of p_g and with NI.

Lemma C.8. *If $p_g > 1$, $\mathcal{Q}^{GSP}$ is decreasing in p_g; $\lim_{p_g \to \infty} \mathcal{Q}^{GSP} = \mathcal{Q}^{NI}$.*

Proof: This result follows directly from Corollary C.1 and (66). In the limit (as $p_g \to \infty$), the market clearance condition in (66) is identical to the one in (56). Therefore, $\lim_{p_g \to \infty} \mathcal{Q}^{GSP} = \mathcal{Q}^{DBT}$. ■

Theorem C.3. *If $p_g > 1$, then, Π_S is decreasing in p_g if $b + \beta < \frac{1}{4}$ and is independent of p_g if $b > \frac{1}{4}$; $\lim_{p_g \to \infty} \Pi_S^{GSP} = \Pi_S^{NI}$.*

Proof: Recall that all monetary payments are transfers from one agent to another. Therefore, we can write the social planner's surplus as

$$\Pi_S = (Mw_{APL} + kMb + B) - \alpha\frac{\mathcal{Q}^2}{n} + \mathbb{E}_\gamma\left[Mu_C(q_{APL}(\gamma)) + kMu_C(q_{BPL}(\gamma))\right]$$

By differentiating Π_S w.r.t p_g, we have:

$$\begin{aligned}\frac{d\Pi_S}{dp_g} &= \frac{\mathcal{Q}}{n/2\alpha}\frac{d\,\mathcal{Q}}{dp_g} + \mathbb{E}_\gamma\left[Mu'_C(q_{APL})\frac{dq_{APL}(\gamma)}{dp_g} + kMu'_C(q_{BPL})\frac{dq_{APL}(\gamma)}{dp_g}\right]\\ &= \mathbb{E}_\gamma\left[\frac{d(kMq_{BPL}(\gamma))}{dp_g}\left(u'_C(q_{BPL}(\gamma)) - u'_C(q_{APL}(\gamma))\right)\right] + \frac{d\,\mathcal{Q}}{dp_g}\mathbb{E}_\gamma\left[u'_C(q_{APL}(\gamma)) - \frac{\mathcal{Q}}{n/2\alpha}\right]\end{aligned}$$

We will show that each term in the RHS above is non-positive; therefore, $\frac{d\Pi_S}{dp_g} \le 0$.

Consider the first term in the RHS above. $q_{BPL}(\gamma) \le q_{APL}(\gamma)$ for all γ. Since $u(q)$ is a concave increasing curve (therefore, $u'(q)$ is decreasing in q), we have that $u'(q_{BPL}(\gamma)) \le u'(q_{APL}(\gamma))$ and holds strictly if $b + \beta < 1/4$. From Lemma C.8 and C.7, for any γ, both $\mathcal{Q}$ and $p_m(\gamma)$ are decreasing in p_g. Therefore, $q_{APL}(\gamma)$ is increasing in p_g and therefore, $q_{BPL}(\gamma)$ is decreasing in p_g. Thus, the first is non-positive.

Consider the second term in the RHS above. $u'_C(q_{APL}(\gamma)) = p_m(\gamma)$. From Lemma C.8, we have that $\frac{d\,\mathcal{Q}}{dp_g} > 0$. Using a similar idea as in the proof of Theorem C.1, $\mathbb{E}_\gamma[p_m(\gamma) - \frac{\mathcal{Q}}{n/2\alpha}] \ge \mathbb{E}_\gamma[p_m(\gamma)\gamma - \frac{\mathcal{Q}}{n/2\alpha}] = 0$. Therefore, the second term is non-positive.

In the limit (as $p_g \to \infty$), we have that the market outcomes are identical and hence $\lim_{p_g\to\infty}\Pi_S^{GSP}(p_g) = \Pi_S^{NI}$. ■

As a consequence, we have the following result.

COROLLARY C.3. *In equilibrium, $\Pi_S^{GSP} \ge \Pi_S^{NI}$ and holds strictly if $b + \beta < \frac{1}{4}$.*

Case 3: Consider an intermediate value of $p_g \in [p_m(1)^{NI}, 1]$: If $p_g = 1$ (resp., $p_g = p_m(1)^{NI}$), the analysis is identical to case 2 (resp., case 1) except at $\gamma = 0$ (resp., $\gamma = 1$), where $p_g = p_m(\gamma)$. If p_g is in the interior of $(p_m(1)^{NI}, 1)$, we have the following. Consider γ s.t. $p_g < p_m(\gamma)$: In this case, $q_g = 0$. (65) can be written as:

$$M(1 - p_m(\gamma)) + kM\min\left\{1 - p_m(\gamma), \frac{b}{p_m(\gamma)}\right\} = \mathcal{Q}\gamma \tag{67}$$

LEMMA C.9. *If $p_g \in (p_m(1)^{NI}, 1)$, there exists $\underline{\gamma}$ such that $p_g < p_m(\gamma)$ iff $\gamma < \underline{\gamma}$. Further, $p_m(\gamma)$ is decreasing in $\gamma \in [0, \underline{\gamma}]$.*

Proof: Observe, from (67) that the LHS is decreasing in $p_m(\gamma)$, while the RHS is strictly increasing in γ. Therefore, $p_m(\gamma)$ is decreasing in γ. If $\gamma = 0$, $p_m(0) = 1$. As γ increases, we have that $p_m(\gamma)$ decreases. We show that as there exists a threshold $\underline{\gamma}$ at which $p_g = p_m(\gamma)$ using contradiction. Suppose not, i.e., suppose $p_g < p_m(\gamma)$ for all γ. Then, the outcome is identical to case 1, where we have shown that the outcome is in turn identical to NI. Therefore, $p_m(1) = p_m(1)^{NI}$. However, recall that $p_g \in (p_m(1)^{NI}, 1)$, which contradicts $p_g < p_m(\gamma)$ for all γ. ■

Therefore, if p_g is in the interior of $(p_m(1)^{NI}, 1)$, there exists $\underline{\gamma}$ s.t. $p_g < p_m(\gamma)$ iff $\gamma < \underline{\gamma}$; otherwise $p_g \geq p_m(\gamma)$. Otherwise, if $\gamma \leq \underline{\gamma}$, $p_g \geq p_m(\gamma)$ and $q_g \geq 0$. The market clearance condition here can be written as:

$$M(1 - p_m(\gamma)) + kM \min\left\{1 - p_m(\gamma), \frac{b}{p_m(\gamma)} + \frac{n}{kM} q_g\right\} = \mathcal{Q}\gamma \tag{68}$$

If $p_g > p_m(\gamma)$, then $q_g = \frac{B}{np_g}$.

Lemma C.10. *If* $p_g \in [p_m(1)^{NI}, 1]$, $\mathcal{Q}^{GSP} \geq \mathcal{Q}^{NI}$.

Proof: This result follows from Corollary C.1 by direct comparison of the market clearance conditions (67), (68) with that in (54). ■

To conclude, $\mathcal{Q}^{GSP} \geq \mathcal{Q}^{NI}$ if $p_g \geq p_m(1)^{NI}$, and $\mathcal{Q}^{GSP} = \mathcal{Q}^{NI}$ if $p_g \leq p_m(1)^{NI}$.

Appendix D: Simulation Procedure for Numerical Experiments

Consider an arbitrary distribution for the yield as follows:

$$\gamma = \begin{cases} \gamma_1, & \text{w.p. } \theta_1; \\ \gamma_2, & \text{w.p. } \theta_2; \\ \vdots & \\ \gamma_N, & \text{w.p. } \theta_N. \end{cases}$$

where $\sum_{i=1}^{N} \theta_i = 1$.

The simulation procedure under DBT is identical to NI except that b is replaced by $b+\beta$, where $\beta = \frac{B}{kM}$.

Algorithm 1: Equilibrium Outcome Under NI

Data: M, k, n, b, α, $G=(\gamma_1, \gamma_2, \ldots, \gamma_N)$, $\Theta=(\theta_1, \theta_2, \ldots, \theta_N)$

```
Function effort
    Input: p̂_m(γ)
    Output: q_e
    q_e ← (1/2α) × SumProduct(p̂_m(γ), Θ, G)        /* q_e^* = E_γ[p̂_m(γ)γ]/2α from (9) */
    return q_e
end
Function demand
    Input: p
    Output: D
    D ← M(1−p) + kM min{b/p, 1−p}                  /* D(p_m) from LHS of (5) */
    return D
end
Function market-price
    Input  : Q
    Output: p_m(γ)
    forall γ ∈ G do
        p_m(γ) ← Unitroot of demand(p) = Qγ.       /* p_m solves (10) */
    end
    return p_m(γ)
end
Function equilibrium
    Output: q_e^NI
    p_m(γ) ← repeat(0, N)                          /* Assign arbitrary values to p_m(γ) */
    repeat
        p̂_m(γ) ← p_m(γ)
        q ← effort(p̂_m(γ))
        Q ← nq                                     /* All farmers exert the same effort */
        p_m(γ) ← market-price(Q)
    until ||p̂_m(γ) − p_m(γ)|| ≤ ε;
    return effort(p_m(γ))
end
```

Algorithm 2: Equilibrium Outcome Under GSP

Data: $M,\ k,\ n,\ b,\ B,\ \alpha,\ G=(\gamma_1,\ \gamma_2,\ldots,\gamma_N),\ \Theta=(\theta_1,\theta_2,\ldots,\theta_N)$

$\mathcal{P}_g \leftarrow$ seq(start $=0$, end $=1$, by $=\epsilon$) `/*` $\mathcal{P}_g$ `is the set of prices we search over to identify the equilibrium support price */`

Function *surplus*

  Input : p_g

  Output: Π_S

  $\mathbf{p}_m(\gamma) \leftarrow$ repeat$(0,\ N)$ `/* Assign arbitrary values to` $\mathbf{p}_m(\gamma)$ `*/`

  repeat

    $\hat{\mathbf{p}}_m(\gamma) \leftarrow \mathbf{p}_m(\gamma)$

    `/*` q_e^* `solves` $q_e = \frac{\mathbb{E}[\hat{p}_m(\gamma)\gamma]+\mathbb{E}\left[(p_g-\hat{p}_m(\gamma))\gamma 1\left(p_g>p_m(\gamma),\gamma q_e<\frac{B}{np_g}\right)\right]}{2\alpha}$ `from` (64) `*/`

    $q_e \leftarrow$ `FixedPoint` of $\frac{1}{2\alpha}\times$ $\Big($`SumProduct`$(\hat{\mathbf{p}}_m(\gamma),\Theta,G)+$`SumProduct`$((p_g-\mathbf{p}_m(\gamma)),\mathbf{1}(p_g>\mathbf{p}_m(\gamma)),\gamma q_e<\frac{B}{np_g}),G,\Theta\Big)$

    $\mathcal{Q} \leftarrow nq_e$

    forall γ *in* G **do**

      if $p_g >$ *market-price*$\left(\mathcal{Q}\gamma-\min\left\{\mathcal{Q}\gamma,\frac{B}{p_g}\right\},\frac{\min\left\{\mathcal{Q}\gamma,\frac{B}{p_g}\right\}}{kM}\right)$ **then**

        `/* If` $p_g>p_m(\gamma)$ `*/`

        $p_m(\gamma) \leftarrow$`market-price`$\left(\mathcal{Q}\gamma-\min\left\{\mathcal{Q}\gamma,\frac{B}{p_g}\right\},\frac{\min\left\{\mathcal{Q}\gamma,\frac{B}{p_g}\right\}}{kM}\right)$

        $q_g,q_m \leftarrow \min\left\{\gamma q_e,\frac{B}{np_g}\right\},\gamma q_e-q_g$

      else if $p_g <$*market-price*$(\mathcal{Q}\gamma,0)$ **then**

        `/* If` $p_g<p_m(\gamma)$ `*/`

        $p_m(\gamma) \leftarrow$ `market-price`$(\mathcal{Q}\gamma,0)$

        $q_g,q_m \leftarrow 0,\gamma q_e$

      else

        `/* If` $p_g=p_m(\gamma)$ `*/`

        $p_m(\gamma) \leftarrow p_g$

        $q_g,q_m \leftarrow$ `UnitRoot` of `demand`$(p_g,\ \frac{n}{kM}q_g)\ =\ \mathcal{Q}\gamma-nq_g,q_e\gamma-q_g$

      end

    end

  until $||\hat{\mathbf{p}}_m(\gamma)-\mathbf{p}_m(\gamma)||\le\epsilon$;

  forall γ *in* G **do**

    $u_C^{APL}(\gamma) \leftarrow$ `Integrate`$(1-x,\ 0,\ 1-p_m(\gamma))$ `/*` u_C^{APL} `from` (1) `*/`

    $u_C^{BPL}(\gamma) \leftarrow$ `Integrate`$\left(1-x,\ 0,\ \min\left\{\frac{b}{p_m(\gamma)},1-p_m(\gamma)-\frac{n}{kM}q_g\right\}\right)$

    `/*` u_C^{BPL} `from` (3) `*/`

  end

  $\Pi_S \leftarrow Mw_{APL}+kMb+B-n\alpha q_e^2+$ `SumProduct`$(G,Mu_C^{APL}(\gamma)+kMu_C^{BPL}(\gamma))$

  `/*` Π_S `from` (7) `*/`

  return Π_S

end

```
Function demand
    Input  : p, q_S
    Output: D
    D ← M(1 − p) + kM min{ b/p, 1 − p − q_S }
    return D
end
Function market-price
    Input  : S, q_S
    Output: p_m
    p_m ← Unitroot of demand(p, q_S) = S.
    return p_m
end
Function equilibrium
    Output: p_g^*
    forall p_g in 𝒫_g do
        Π_S(p_g) ← surplus(p_g)
    end
    p_g^* ← which.max(Π_S(p_g)) for p_g in 𝒫_g.          /* p_g^* = arg max_{p_g ∈ 𝒫_g} Π_S(p_g) */
    return p_g^*
end
```